Optimization of helical movement of magnetic axis
in LHD-type planar-axis stellarator

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The Large Helical Device (LHD) is a stellarator with super-conducting magnets (major radius: \( R = 3.9 \text{ m} \)). During 15 years of operation, LHD has produced lots of important experimental results, which contribute to the development of an alternative type of fusion reactor concept other than tokamak. Two of important performances are stable discharges of 5% volume-averaged beta and high-density operations above \( 10^{20} \text{m}^{-3} \). The magnetic field configuration of LHD is of a conventional stellarator, where a non-axisymmetric element of the field is a simple helical structure of the poloidal mode \( m = 2 \). Other non-axisymmetric elements are small, which is in contrast to many advanced stellarators that have large amplitudes of configuration elements for multi-harmonics. The shape of the magnetic axis in LHD is almost a circular ring, which defines a word "planar-axis stellarator". However, it was found [1] that a small excursion of the magnetic axis from a circular ring, which is called "helical axis", produces a large difference of the confinement characteristics in different settings of magnetic configuration in LHD experiments (magnetic axis shift). In this paper, we study systematically the effects of helical axis structure introduced in LHD-type planar-axis stellarator configuration.

As a mathematical tool for describing the three-dimensional structure of magnetic configurations of stellarators, Fourier modes expressions of magnetic surfaces with two angle variables are widely used.

\[
R(\theta, \phi) = \sum r(m, n) \cdot \cos(m\theta - n\phi)
\]
\[
Z(\theta, \phi) = \sum z(m, n) \cdot \sin(m\theta - n\phi)
\]

where \( \theta \) and \( \phi \) are two (poloidal and toroidal) angle parameters mapped on the magnetic surface. \( R \) and \( Z \) define a point on the magnetic surface in a cylindrical coordinate system. In this paper, we use a different type of magnetic surface expressions described in the following formula [2].
where \( i \) is the imaginary unit and other numbers are real. The advantage of this expression is much simpler correspondences between coefficients \( \Delta_{mn} \) and geometrical quantities of the shape of magnetic surface.

Figure 1 shows the comparison of geometrical elements of the last closed magnetic surface (LCMS) of LHD and Wendelstein 7-X (W7-X) in vacuum configurations as typical examples of a planar-axis and a non-planar-axis stellarator. For LHD, the configuration with the magnetic axis position \( R_{\text{ax}} = 3.6 \text{ m} \) is selected. Absolute values of coefficients \( \Delta_{mn} \) are plotted in logarithmic scale for the range \(-3 \leq m \leq 3 \) and \(-3 \leq n \leq 3 \). The tallest one \( \Delta_{00} \) is for the averaged minor radius \( a \) and all coefficients are normalized by \( a \). So \( \Delta_{00} \) appears as 1.0 in these figures. The tall blue column \( \Delta_{11} \) in Fig. 1(a) is for the rotating elliptical cross-section, which is the basic helical structure. The shorter blue column \( \Delta_{11} \) in Fig. 1(a) is for the helical movement of plasma cross-section (similar to the movement of the magnetic axis), which is larger for W7-X in Fig.1(b). LHD configuration has only these elements as fundamental components and other coefficients are very small (less than 0.017 \([1/10^{1.5}]\) times \( a \)). On the other hand, W7-X configuration has about three times more elements as fundamental components. The coefficients of \( \Delta_{11} \) and \( \Delta_{1,-1} \) are more significant than LHD, which create larger helical movement of plasma cross-section. \( \Delta_{1,-1} \) is for the rotating crescent shape which is general characteristic feature for many advanced stellarators. W7-X has also two \( n = 0 \) components \( \Delta_{-10} \) and \( \Delta_{20} \), which are axisymmetric crescent and elliptical shape. Two elements \( \Delta_{22} \) and \( \Delta_{32} \) with \( n = 2 \) are indispensable components too.

In order to clarify the effect of non-planar axis structure of stellarator, we calculated
vacuum equilibria of LHD with different $\Delta_{11}$ keeping other elements unchanged. Figure 2 shows the variations of the effective helical ripples $\varepsilon_{\text{eff}}$ (in the logarithmic scale) as functions of $\Delta_{11}$ (in relative value to $a$). $\varepsilon_{\text{eff}}$ [3] are calculated at the radii of two magnetic surfaces ($(r/a) = 1/3$ and $2/3$) for the evaluation of the neo-classical transport. The best performance of confinement is obtained for $\Delta_{11} = 0.1$. Three arrows are plotted in the figure, which indicate the level of helical excursion for three LHD configurations that are used in most of the experiments (#1: $R_{ax} = 3.6$ m, #2: $R_{ax} = 3.75$ m, #3: $R_{ax} = 3.9$ m). It was shown in [1] that the confinement properties of LHD configurations are basically determined by this simple geometric term $\Delta_{11}$. The sign of $\Delta_{11}$ does not mean the direction of the rotation of helical excursion. The rotation is always in the same direction, which is the same movement as the rotation of the elliptical cross section. When they are in the opposite direction, no effect on confinement is given by the helical excursion. The positive sign of $\Delta_{11}$ means that the excursion starts from the position of a larger major radius than the averaged one at the toroidal angle of the vertically elongated cross-section.

The MHD stability also strongly depends on this geometric term $\Delta_{11}$. Figure 3 shows the relative specific volume at the plasma boundary $SV$-edge and magnetic well depth $MW$ as functions of $\Delta_{11}$. $SV$-edge and $MW$ are defined as following.

$$V_{sp} = \frac{dV}{d\Psi}, \quad SV\text{-edge} = \frac{V_{sp}(r = a)}{V_{sp}(r = 0)}, \quad MW = -\frac{V_{sp}(r = r_{\min}) - V_{sp}(r = 0)}{V_{sp}(r = 0)}$$

Although the edge region of LHD configuration is always in the magnetic hill, this physical quantity of edge specific volume is good indication for the MHD stability in high beta discharges. As the plasma beta is increased, $SV$-edge goes down and the critical stability is obtained for whole region of plasma confinement. In the process of configuration optimization,
this SV-edge can be used as one of the benefit terms for the MHD stability. Figure 3 shows again the fact that it is difficult to obtain a good confinement and the MHD stability simultaneously in LHD configuration.

For the simple configuration of LHD given in Fig. 1(a), a large helical excursion give no benefit to either in confinement nor in MHD stability. The situation is different for W7-X. Figure 4 shows the same dependence of effective helical ripple on $\Delta_{11}$ for the configuration of W7-X. It has the minimum of effective ripple at larger value of $\Delta_{11}$. In this case, a large excursion of the cross section contributes to improve the confinement.

Fig. 3 Specific volume at plasma edge and magnetic well as functions of helical excursion of the cross section in LHD configuration.

Fig. 4 Effective helical ripples at $(r/a) = 1/3$ and $2/3$ as functions of helical excursion of the cross section in W7-X configuration.

References