Effect of Impurity Toroidal Viscosity on Offset Toroidal Rotation

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Introduction

One of essential differences between tokamak and helical is the geometrical symmetry [1]. Recently, changes in plasma properties induced by the application of non-axisymmetric field to tokamak attract strong interests. Especially, observation of offset toroidal rotation [2] is important for stabilizing RWM even if it is small. Analytical offset toroidal rotation formula derived in 2011 has been extended to include effect of impurity toroidal viscosity.

Offset Toroidal Rotation due to Bulk Ion Toroidal Viscosity

Ambipolarity is ensured in the axisymmetric tokamak, irrespective of electric field. If the symmetry braking occurs, the ambipolarity of the particle flux is broken. Then the radial electric field (electrostatic potential) is adjusted to satisfy ambipolarity (non-ambipolar flux=0). Assuming impurity toroidal viscosity, which is in Pfirsh Schluter regime, ambipolarity (non-ambipolar flux=0) condition is given by

\[ \langle \mathbf{B}_t \cdot \nabla \cdot \mathbf{\Pi}_i \rangle = 0. \]

Shaing [3] gave following expression for this neoclassical toroidal viscosity (NTV) \( (\mathbf{U}_i = dV/d\psi) \).

\[ \langle \mathbf{B}_t \cdot \nabla \cdot \mathbf{\Pi}_i \rangle = e_i B_r B_p \Gamma_{ri} = -e_i n_i B_r B_p q_i^2 \tau_{ii} \left( \frac{\delta T}{e_i B_r} \right)^2 \left( \frac{\epsilon \pi}{\lambda} \right)^{3/2} \left[ \lambda_{1i} \left( \frac{P_i^l}{P_i} + e_i \Phi^l \right) T_i^l \right] + \lambda_{2i} T_i^l \] (1)

Therefore non-ambipolar flux=0 condition, \( \langle \mathbf{B}_t \cdot \nabla \cdot \mathbf{\Pi}_i \rangle = 0 \) gives \( \frac{d\Phi}{d\psi} + \frac{1}{e_i n_i} \frac{dP_i}{d\psi} = -\frac{\lambda_2}{e_i \lambda_1} \frac{dT_i}{d\psi} \).

To get an analytic expression for the offset toroidal rotation, we use the 0-th order radial force balance equation to see a relation among the offset toroidal rotation, the thermodynamic force, and the residual poloidal flow.

\[ \mathbf{u}_{i0} \cdot \nabla \zeta = - \left[ \frac{d\Phi}{d\psi} + \frac{1}{e_i n_i} \frac{dP_i}{d\psi} \right] + q \mathbf{u}_{i0} \cdot \nabla \theta \] (2)

where \( q \) is the safety factor. By using an analytic expression for the residual poloidal rotation by Kim [6], \( \mathbf{u}_{i0} \cdot \nabla \theta = -\frac{K_1 F(B \cdot \nabla \theta)}{e_i \lambda_1} \frac{dT_i}{d\psi} \), an analytic expression for the offset toroidal rotation of the bulk ion for NTV, \( u_{i\zeta 0} = R \mathbf{u}_{i0} \cdot \nabla \zeta \) is given by Kikuchi [4], [5] as,

\[ u_{i\zeta 0} = R \left[ \frac{\lambda_2}{e_i \lambda_1} - \frac{q K_1 F(B \cdot \nabla \theta)}{e_i \lambda_1 \langle B^2 \rangle} \right] \frac{dT_i}{d\psi} \] (3)

In the large aspect ratio cylindrical plasma, offset toroidal rotation is given as follows,

\[ u_{i\zeta 0} = \frac{3.54 - K_1}{e_i B_\theta} \frac{dT_i}{dr} \] (4)
Since measurement of toroidal rotation is made using impurity toroidal rotation, the expression of the offset toroidal rotation of the impurity is required for comparison to the experiment. We use a formula
\[ u_{I\xi_0} - u_{i\xi_0} = -1.5K_2 \frac{dT_i}{dr} \] [1] to obtain following formula.
\[ u_{I\xi_0} = \frac{3.54 - 1.5K_2 - K_1 dT_i}{eZ_iB_\theta} \frac{dT_i}{dr} \tag{5} \]

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In a multi species plasma, the zero non-ambipolar flux condition is given as,
\[ \sum_a e_a \Gamma^a_{na} = 0 \tag{6} \]
, where
\[ \Gamma^a_{na} = \frac{\langle B_t \cdot \nabla \cdot \Pi_i \rangle}{e_a \psi' \phi'} \tag{7} \]

Here, \( B \cdot \nabla \zeta = \phi' \) and \( B \cdot \nabla \theta = -\psi' \). Considering the electron viscosity is small, condition for zero non-ambipolar flux \( \sum_a e_a \Gamma^a_{na} = 0 \) is given as,
\[ \langle B_t \cdot \nabla \cdot \Pi_i \rangle + \langle B_t \cdot \nabla \cdot \Pi_i \rangle = 0 \tag{8} \]

Explicit form of the impurity toroidal viscosity is given in the Appendix. Here we use successive approximation to obtain first order correction to offset toroidal rotation. We expand offset toroidal rotation as \( u_{\xi} = u_{\xi_0} + u_{\xi_1} + ... \). 0-th and 1-st order equations are given as follows,
\[ \langle B_t \cdot \nabla \cdot \Pi_i \rangle_0 = 0 \tag{9} \]
\[ \langle B_t \cdot \nabla \cdot \Pi_i \rangle_1 + \langle B_t \cdot \nabla \cdot \Pi_i \rangle_0 = 0 \tag{10} \]

Obviously, the solution for (9) is given by (4). Solution to the 1-st order equation (10) may be given by approximating \( u_{\xi_1} \sim E_{r1}/B_p = -\frac{d\Phi_1}{dV} \frac{dV}{dr}/B_p \) and using the equation (1) to obtain,
\[ \langle B_t \cdot \nabla \cdot \Pi_i \rangle_1 = p_i \tau_{ii} \left( \frac{\delta}{R} \right) \left( \frac{E}{2} \right)^{3/2} r'(V)B_t \lambda_{1i} u_{\xi_1} \tag{11} \]

Therefore, 1-st order toroidal flow due to impurity NTV, \( u_{\xi_1} \) is given by,
\[ u_{\xi_1} = -\frac{dV}{dr} \frac{\pi^{3/2} R^2 \langle B_t \cdot \nabla \cdot \Pi_i \rangle_0}{p_i \tau_{ii} e^{3/2} B_t \lambda_{1i}} \tag{12} \]
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Appendix: Impurity NTV in PS regime

Neoclassical toroidal viscosity (NTV) in Pfirsh-Schluter regime for impurity is given in Hamada coordinates by Shaing [8] as,

$$\langle B_i \cdot \nabla \cdot \Pi_i \rangle = 3 \left( \frac{B_i \cdot \nabla \theta}{B \partial \theta} \right) (\mu_{11} u \cdot \nabla \theta + \mu_{12} \frac{2q}{5p_i} \cdot \nabla \theta) + \left( \frac{B_i \cdot \nabla \theta}{B \partial \theta} \right) (\mu_{11} u \cdot \nabla \zeta + \mu_{12} \frac{2q}{5p_i} \cdot \nabla \zeta)$$

(13)

Here original paper by Shaing [8] includes $p_I \tau_H$, which is not necessary. Viscosity coefficients are given by Hirshman-Sigmar [7] as follows,

$$\mu_{a1} = K_{11}^a \mu_{a2} = K_{12}^a - \frac{5}{2} K_{11}^a \mu_{a3} = K_{22}^a - 5K_{12}^a + \frac{25}{4} K_{11}^a$$

(14)


$$K_{ij}^a = \mu_a \tau_{aa} q_{ij}^a$$

(15)

$$t_{i1}^a = \frac{q_{11}^a}{Q_a}, t_{i2}^a = \frac{7 (q_{11}^a + q_{01}^a)}{2 Q_a} = t_{i1}^a, t_{i2}^a = \frac{49 (q_{11}^a + q_{00}^a + 2q_{01}^a)}{4 Q_a}$$

(16)

$$Q_a = \frac{2}{5} (q_{00}^a q_{11}^a - q_{01}^a q_{01}^a)$$

(17)

$$q_{ab}^0 = \frac{3 + 5x_{ab}^2}{(1 + x_{ab}^2)^{3/2}}, q_{ab}^0 = \frac{3}{2} (1 + x_{ab}^2)^{3/2}, q_{ab}^1 = \frac{35x_{ab}^6 + 77x_{ab}^4 + 185x_{ab}^2 + 51}{16(1 + x_{ab}^2)^{7/2}}$$

(19)

$$r_{aa}^0 = \frac{1}{\sqrt{2}}, r_{aa}^1 = \frac{3}{2\sqrt{2}}, r_{aa}^1 = \frac{15}{4\sqrt{2}}$$

(20)

In case $T_i \approx T_L$, $x_{aa} = 1, x_{ii}^2 \approx \frac{m_i}{m_f} << 1, x_{ii}^2 \approx \frac{m_i}{m_f} \gg 1$ when i is deuterium and I is Carbon, since $x_{ab} = \sqrt{v_{Th}/v_{Ta}}.$

$$q_{ii}^0 = \frac{15}{4\sqrt{2}}, q_{ii}^0 = 2\sqrt{2}, q_{ii}^1 = \frac{265}{16\sqrt{2}}$$

(21)

Since $x_{ii}^2 \gg 1$, we have

$$q_{ii}^0 = 5 x_{ii}^2, q_{ii}^1 = 21 x_{ii}^3, q_{ii}^1 = \frac{35}{x_{ii}^2}$$

(22)

From (18), we have $q_{ij}^i = q_{ij}^i - r_{ij}^i + q_{ii}^i/\alpha$, where $\alpha = n_iZ_i^2/n_iZ_i^2$ is called impurity strength parameter. And,

$$q_{ij}^0 = \frac{3}{\sqrt{2}} + \frac{5}{\alpha x_{ii}^2}, q_{ij}^0 = \frac{9}{4\sqrt{2}} + \frac{21}{2\alpha x_{ii}^2}, q_{ij}^1 = \frac{205}{16\sqrt{2}} + \frac{35}{\alpha x_{ii}^2}$$

(23)

$$Q_I = f(\alpha, x_{ii}) = \left( \frac{6}{5\sqrt{2}} + \frac{2}{\alpha x_{ii}^2} \right) \left( \frac{205}{16\sqrt{2}} + \frac{35}{\alpha x_{ii}^2} \right) - \frac{2}{5} \left( \frac{9}{4\sqrt{2}} + \frac{21}{\alpha x_{ii}^2} \right)^2$$

(24)
So, $Q_I = \frac{267}{40} + O(1/\alpha x_I)$. Taking the leading order for $Q_I$, we obtain,

$$t'_{11} = \frac{40}{267} \times \frac{205}{16\sqrt{2}} = 1.357$$ (25)

$$t'_{12} = \frac{7}{2} \times \frac{40}{267} \left( \frac{205}{16\sqrt{2}} + \frac{9}{4\sqrt{2}} \right) = 5.586$$ (26)

$$t'_{22} = \frac{49}{4} \times \frac{40}{267} \left( \frac{205}{16\sqrt{2}} + \frac{3}{\sqrt{2}} + \frac{2 \times 9}{4\sqrt{2}} \right) = 26.26$$ (27)

If we define normalized viscosity $\hat{\mu}_I = \mu_I / p_I \tau_{II}$, we obtain,

$$\hat{\mu}_{I1} = 1.357, \hat{\mu}_{I2} = 2.193, \hat{\mu}_{I3} = 6.81$$ (28)

If the magnetic field variation in poloidal and toroidal directions is given as, $B = B_0 [1 - \varepsilon \cos \theta - \delta \cos (m\theta - n\zeta)]$ (29), we have $(\partial B / \partial \theta) / B = \varepsilon \sin \theta + m\delta \sin (m\theta - n\zeta)$. Using the relation $B_t = \phi' \nabla V \times \nabla \theta$ in Hamada coordinates, we obtain,

$$\mathbf{B} \cdot \nabla B = -\frac{\phi'}{2} n m \delta^2$$ (30)

$$\mathbf{B} \cdot \nabla B = \frac{\phi'}{2} n^2 \delta^2$$ (31)

Here, $\phi' = d\phi / dV = \mathbf{B} \cdot \nabla \zeta \sim B_t / R \sim B / R$. Inserting these expression, NTV in PS regimes becomes,

$$\langle \mathbf{B}_t \cdot \nabla \mathbf{I} \rangle = \frac{3}{2} p_I \tau_{II} \phi' n \delta^2 \left[ \hat{\mu}_{I1} U_I + \hat{\mu}_{I2} \frac{2q_I}{5p_I} \right] \cdot [n \nabla \zeta - m \nabla \theta]$$ (32)

References


