Magnetic topology and flow in helical Reversed Field Pinch (RFP) configuration from MHD simulations

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Introduction The RFP is a toroidal magnetic configuration for fusion plasma confinement. Theoretical studies of the configuration, confirmed by experimental phenomenology in several aspects, strongly relied on the numerical solution of the equations of the visco-resistive magnetohydrodynamic model [1]. The results showed the possibility for the configuration to evolve from a state characterized by the action of several magnetohydrodynamic resistive kink-tearing modes (MHD modes) to a state endowed with a high level of helical symmetry due to the action of a single dominant MHD mode (quasi-helical RFP states, so called Quasi Single Helicity, QSH, in contrast with the Multiple Helicity MH states) [2], [3]. The symmetry of the magnetic field is transmitted to its field lines which lie, in the limit of perfect symmetry, on nested helical magnetic surfaces. The study of their topology can provide information to understand the experimental observations and to drive the plasma toward better confinement properties. In MH states the overlapping of several magnetic islands of similar amplitudes associated with MHD modes produces chaotic behaviour of the magnetic field lines; in QSH the presence of a dominant mode allows for conserved magnetic surfaces. This is particularly clear when the dominant mode, in the process of its nonlinear saturation, forms a Single Helical Axis (SHAx) state [4], [5] characterized by the absence of magnetic islands and featuring Internal Transport Barriers [7].

In this paper we study the resilience to chaos of QSH-SHAX states generated by different MHD dominant modes. The feature of chaos healing by dominant mode’s separatrix expulsion is once again confirmed as a dominant effect, but other features like the choice of dominant mode (resonant or non-resonant mode) and the relative amplitude of secondary modes are found to impact on chaos healing properties.

Numerical tools The numerical simulation of RFP is based on the result from the non-linear spectral 3D MHD code SpeCyl, which solves the visco-resistive MHD equations in cylindrical geometry, reproducing toroidal periodicity with aspect ratio $R/a=4$ by imposing periodical boundary conditions on the axial variable $z$. SpeCyl provides the value of the
magnetic field vector in space: these data represent the input for the field line tracing code NEMATO [6]. The code calculates the magnetic field lines trajectory solving the magnetic field lines equation on a 3D logical grid (which can represent a generic curvilinear geometry). It uses a volume preserving algorithm, which, together with the interpolation scheme for points off the grid, ensures the preservation to numerical errors of the condition $\nabla \cdot \mathbf{B} = 0$. Magnetic topology studies are performed examining Poincaré plots that show the intersections with selected surfaces of section of field lines followed for long enough spatial length.

**Figure 1:** a) Radial profile of the safety factor at $t=0\tau_A$ and at the times studied. The $(1,-8)$ MHD mode is initially marginally non resonant with respect to this profile. b) Temporal evolution of the most important MHD modes: the phase $[500:1500]\tau_A$ is a Quasi Single Helicity state generated by the non-resonant mode $(1,-8)$; the phase $[2000:3000]\tau_A$ is a QSH generated by the resonant mode $(1,-10)$. In the interval $[1500:2000]\tau_A$ there is a Multiple Helicity phase. The symbol "&" shows when separatrix is expelled. Yellow areas mark the times of the analysis presented in this paper.

**SpeCyl numerical simulation** We will consider a SpeCyl simulation characterized by constant Lundquist number (ratio between resistive diffusion time and Alfvén time) $S = 3 \cdot 10^4$ (resistivity increases sharply in the edge) and magnetic Prandtl number (ratio between resistive and viscous diffusion time) $P=1000$ (viscosity is uniform in the domain). A wide spectrum of MHD modes with $0 \leq m \leq 4$ ($m$ being the poloidal mode number) is used. The initial axisymmetric equilibrium (whose q-profile is shown in figure 1a) is perturbed by three MHD modes with $m=1$ and toroidal mode number $n=-8$ (marginally non-resonant mode), $n=-9$, $n=-10$ (resonant modes). This kind of simulation may schematically represent the final stage of discharge formation when achieving the (toroidal) magnetic field reversal typical of the RFP. With the choices above, clear long lasting phases of QSH characterized by ratio of dominant to secondary modes similar to experimental observations are easily obtained.

The temporal evolution of the MHD modes energy, figure 1b, shows two phases: the first emergence of the strongly unstable marginally non resonant $(1,-8)$ mode is followed by the establishment of the $(1,-10)$ mode as dominant one after $t=1500\tau_A$ and until the end of the simulation. The topology of the flux surfaces associated with the dominant MHD mode is easily obtained by the helical flux function of helicity $h, \chi_h = m \psi_{pol} + n/R_0 \phi_{tor}$, that in helical symmetry obeys $\mathbf{B} \cdot \nabla \chi_h = 0$. The $(1,-8)$ mode does not form any island (separatrix) all along
its evolution; the \((1,-10)\) mode, resonant since the beginning, expels its separatrix at \(t=2350\tau_A\).

**Results** Before studying the magnetic topology in the two phases highlighted above, let’s introduce two physical quantities useful in discussing experimental observations about RFP electron Internal Transport Barriers [7]: the first is the difference in magnetic energy between the dominant and the most important secondary mode. We here define a state *resilient* if magnetic chaos does not emerge despite low energy difference between dominant and secondary modes. The second quantity is the helical safety factor \(q_h\), which gives the number of toroidal turns that a field line carries out for a single poloidal turn around the helical axis; this has been found experimentally to display a shear reversal at the barrier foot [7].

![Figure 2: a) Poincaré plot taken at \(z=0\), revealing the \(m=1\) poloidal periodicity of the mode. b) Poincaré plot taken at \(\theta=0\), revealing the \(n=-8\) axial periodicity. In both plots perfectly conserved magnetic surfaces are obtained by NEMATO. c) Helical safety factor calculated considering the \(-n=-8\) modes as dominant helical perturbation. The value of the flow shear [7] in the direction parallel to \(\nabla \times B\) is plotted in red.](image)

Consider the QSH state generated by the non resonant mode: at \(t=1400\tau_A\) the energy difference between \((1,-8)\) and \((1,-9)\) is \(\Delta[\log W_M]=0.6\), resulting in a ratio of 2 between the corresponding magnetic fields. On both plots in figure 2 the reader will notice a topology characterized by a simple helical deformation of the plasma column, and perfectly conserved magnetic surfaces. Despite its significant amplitude the resonant mode \((1,-9)\), together with the other secondary ones, is not able to produce chaos. Thus, this state is strongly resilient to chaos: in fact further analyses indicate that this property is found even at negligible energy difference (larger secondary modes amplitude). The helical \(q_h\) profile, figure 2c, exhibits a wide low shear (without shear reversal) region around the core.

Consider now the phase during which the resonant mode \((1,-10)\) is dominant, soon after the separatrix expulsion (occurred at \(t=2350\tau_A\)). At \(t=2400\ \tau_A\) the energy difference is much higher than that in the previous case (\(\Delta[\log W_M]=2.7\)) corresponding to a ratio of 23 between the magnetic components. NEMATO detects (see figure 3a) a wide chaotic region and some conserved magnetic surfaces. Despite the very high energy difference, the secondary mode is still influencing the system indicating the weakness and the low chaos resilience of the SHAx state generated by the resonant mode.
An interesting observation can be highlighted: conserved structures start to appear in the core region where the \( q_h \) profile is flat, namely for \( \rho < 0.3 \) and around \( \rho \sim 0.6 \) (shadowed areas in figure 3b). Later on, chaos healing proceeds (eventually leading to a perfectly ordered SHAx state like in figure 2a). This is accompanied by both a small variation (decrease) in modes’ energy and increase in \( q \) flatness (close to the 1/13 resonance), similarly to [8] for the resonant \( n=-9 \) case.

**Figure 3** (a) Poincaré plot at \( z=0 \). Red lines refer to conserved magnetic surfaces, black lines to chaotic field lines regions. (b) Helical safety factor profile: grey areas represent the values of the radius \( \rho \) where conserved magnetic surfaces are present. At this time the flow shear [7] exhibit a clear maximum in correspondence of the \( q \) profile extreme.

**Conclusions** Our numerical analysis shows that a SHAx state generated by a resonant mode (with respect to the axisymmetric \( q \) profile) is not as resilient to chaos as the one generated by a non resonant mode (lower toroidal periodicity). More in general, the different degree of chaos resilience observed in our cases suggests the presence of a hierarchy among the dominant modes, which rules the actual impact of secondary modes in producing chaos. Our results can be useful in relation to RFX-mod experiment whose SHAx states are presently diagnosed to be associated with the first resonant MHD mode. The possibility to let, instead, a non-resonant mode build up the experimental SHAx state is currently under investigation. A more extended study of the helical safety factor profile could provide possible links with Refs. [9], [10]. Those papers discuss the impact of \( q \) flatness and of its “distance” from resonant values on chaos formation/healing. Concerning the flow shear features, which may impact on additional transport mechanisms [7], we find that higher toroidal periodicities (resonant modes) provide SHAx states where the flow shear is stronger.

**References**