Nonlinear gyrokinetic simulations of microtearing mode turbulence

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First non-linear gyrokinetic simulations of microtearing mode turbulence have recently been reported for a high-collisionality NSTX discharge (B_t=3.5kG, I_p=0.7MA, R/a=0.82/0.62 m) [1] and independently for conventional tokamak parameters [2]. Using the Eulerian gyrokinetic code GYRO [3] with finite collisionality (ν_{ei}=1.26 c/a, Z_{eff}=2.9) and electromagnetic perturbations (δA_||, β_e=8πn_eT_e/B_0²=8.8%), it was shown for NSTX at r/a=0.6 (where only microtearing modes are linearly unstable) that relatively fine radial grid spacing was required to sufficiently resolve fluctuations and demonstrate monotonically decaying turbulence spectra [1]. Fig. 1 shows a snapshot of the resulting density (δn/n_0) and magnetic field (δB_r) fluctuations at a fixed toroidal angle in real space (R,Z) coordinates. Narrow

Fig. 1. (left) Normalized density perturbations and (right) shear magnetic field perturbations.
radial density perturbations are apparent on the outboard side, which correspond to the narrow resonant parallel current perturbations responsible for destabilizing the microtearing mode. To resolve these perturbations, the simulation used 400 radial grid points [and 8 complex toroidal modes with toroidal mode spacing $\Delta n=5$ or $(k_0\rho_s)_{\text{min}}=0.105$, $k_0=nq/r$, $q=1.69$], with a spacing $\Delta x=0.2\rho_s$ that is capable of resolving the high order rational surfaces ($n_{\text{max}}=35$, $k_0\rho_{s,\text{max}}=0.735$) separated by $\Delta r_{\text{rat},\text{min}}=1/n_{\text{max}}q'=1/k_0_{\text{min}}s=0.78\rho_s$ ($s=r/q\cdot dq/dr=1.75$).

An additional simulation using 16 toroidal modes and 540 radial grid points ($\Delta n=3$, $\Delta x=0.15\rho_s$), requiring $\sim250,000$ cpu-hrs on the Oak Ridge “Jaguar” Cray XT5, exhibits very similar behavior, illustrating, at least qualitatively, the microtearing turbulence is resolved.

While local peak density perturbations of $\delta n/n \sim 2\%$ occur around the entire poloidal circumference, the $\delta B_r$ perturbations are strongly ballooning. The structure of $\delta B_r$ is spatially broad on the outboard midplane with local instantaneous values as large as 30 Gauss ($\sim 1\%$ of the vacuum field strength, $B_0=3.5\text{kG}$). Using both $\delta n$ and $\delta B_r$ from multiple time slices, calculations predict 1-2° of instantaneous polarimetry phase shift ($\sim 0.3°$ rms) dominated by $\delta B_r$, which may be measurable by a new polarimetry system to be installed on NSTX [4]. Furthermore, the outboard density perturbations may be measurable by the “high-k” coherent scattering diagnostic [5] which is sensitive to density perturbations with $k_r>k_0$.

The magnetic “flutter” from the strong $\delta B_r$ fluctuations is responsible for $\sim 98\%$ of the total electron thermal transport, $\chi_{e,\exp} \approx 5-8$ $\rho_sc^2/a=6$ $\text{m}^2/\text{s}$, which is in the experimental range $\chi_{e,\exp} \approx 5-8$ $\text{m}^2/\text{s}$. Estimates using the spectrum of rms saturated amplitudes ($\langle|\delta B_r/B|\rangle^{1/2} \sim 0.15\%$) predict that island widths, $w_{\text{island}} = 4\sqrt{\delta B_r/B \cdot rR/\text{ns}}$, should be larger than the minimum separation in rational surfaces $w_{\text{island}}/\delta r_{\text{rat}} \leq 8$, $\delta r_{\text{rat}}\approx\Delta n/n^2q'$ [1], satisfying the Chirikov overlap criteria [6]. Poincare surface-of-section plots, formed by integrating 100 field line trajectories for 3000 poloidal transits, confirm global stochasticity throughout the entire simulation domain [7]. From the integrated trajectories the magnetic diffusivity is calculated $D_M \approx 6\times10^{-7}$ $\text{m}$, from which we infer a magnetic correlation length $L_c \approx 0.25$ $\text{m}$ much shorter than the electron mean free path $\lambda_{\text{mfp}}=12.5$ $\text{m}$. The resulting collisionless test particle stochastic transport model, $\chi_{e,\text{st}} = 2\sqrt{2/\pi} \cdot f_p D_m v_{Te}$, [7] agrees to within $\sim 20\%$ of the simulated transport.
Fig. 2 shows the predicted transport from simulations that vary normalized electron temperature gradient \((a/L_{Te})\), collisionality, beta, and E×B shear \((\gamma_E)\), defined in [8]) using the smaller of the above grids (~100,000 cpu-hrs/sim). The electron thermal transport is very “stiff”, varying ±~100% for ±20% changes in \(a/L_{Te}\), suggesting it would be useful to characterize the scaling of the threshold gradient. Most noteworthy is that the predicted collisionality scaling, \(\chi_{e,sim} \sim \nu_e^{1.1}\), is roughly consistent with global energy confinement scaling in both NSTX and MAST [9], \(\Omega_i\tau_e \sim \nu_s^{-(0.82-0.95)}\). This gives strong evidence for the importance of microtearing modes in determining confinement scaling in NSTX, as no other micro-instability is predicted to be unstable for these parameters. However, including the experimental value of E×B shear, \(\gamma_{E,exp} \approx 0.17 c_s/a\) (comparable to the maximum linear growth rate) reduces the predicted transport to negligible values. Given the stiff behavior with \(a/L_{Te}\), an additional \(\gamma_E\) scan was run with \(1.2 \times a/L_{Te,exp}\). The transport at \(\gamma_{E,exp}\) is now larger, but still

\[\text{Fig. 2. Predicted } \chi_e \text{ vs. (a) } a/L_{Te}, (b) \nu_e (c_s/a) \text{ (log-log scale), (c) } \beta_e \text{, and (d) } E \times B \text{ shear rate, } \gamma_E \text{ (c}_s/a). \text{ The shaded regions indicate experimental values with uncertainties.}\]
3-4× smaller than experiment. While the radial resolution in these simulations is marginally sufficient to distinguish the narrow resonant parallel current perturbations ($\Delta j \leq 0.3 \rho_s$) responsible for the microtearing instability, the quantitative linear growth rates are smaller than highly resolved linear simulations ($\Delta x \approx 0.03 \rho_s$ for $k_\theta \rho_s,_{\text{max}}$), especially as marginal stability is approached (e.g. reduced $a/L_T e$, $v_e$, or $\beta_e$). As a result, with improved non-linear resolution the magnitude of transport in Fig. 2 might be increased sufficiently to recover experimental transport with $\gamma_{E,\text{exp}}$. Another uncertainty in predicting quantitative transport stems from the limited resolution in toroidal modes ($k_\theta$). Fig. 3 shows the fractional transport spectra for three values of $\gamma_E$. While there is a well defined peak at $k_\theta \rho_s \approx 0.21$ with no E×B shear, the tail is very weakly decaying. It is possible that additional transport from $k_\theta \rho_s >0.73$ can contribute non-negligibly to the total. Indeed, the transport from the higher resolution case is ~25% larger. This effect is exacerbated as $\gamma_E$ approaches the experimental value where there is no clear peak in transport. Simulations with higher binormal resolution are required to study this effect further. Finally we note that these simulations were run in the local limit, i.e. with no variation in equilibrium profiles ($q$, $a/L_T e$, etc…). Including profile variation, which is likely important at large $\rho_s/a \approx 1/120$ in NSTX, may produce some non-local modification of these results.

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