Heavy Particle Modes and Signature of the I-Regime

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Introduction

Recent investigations of the I-Regime by the Alcator C-Mod machine have brought to light features of this regime that we propose to explain by the excitation of a new kind of heavy particle (impurity) mode [1, 2] at the edge of the plasma column. This mode involves both density and magnetic field fluctuations. The transport of heavy particles that is produced by it is outward, while the main plasma ions are transported inward. This can lead to a high degree of plasma purity, a condition necessary to reach ignition in fusion burning plasma. These features are consistent with those of the observed mode having a frequency about 200 kHz and with the finding [3, 4, 5, 6, 7, 8] that, in the I-Regime, impurities are confined at the edge. The proposed mode has a phase velocity in the direction of the electron diamagnetic velocity, a feature was found first theoretically and that has been observed later experimentally [9].

I-mode and Its Transport Properties

Most features of the mode can be described in a plane plasma configuration where the equilibrium magnetic field \( B \simeq B e z \). The plasmas consist of the electrons with density \( n_e(x) \), the main ion population with density \( n_I(x) \) and \( Z = 1 \) and a heavy ion (impurity) population with density \( n_I(x) \) and mass number \( A_I \). We consider the electrostatic perturbations with \( \hat{E} = -\nabla \hat{\phi} \), which are justified by the estimate given in Ref. [2], and the modes of the form \( \hat{\phi} = \hat{\phi}(x) \exp(-i\omega t + ik_z z + ik_y y) \). The relevant longitudinal phase velocities are in the range \( \nu_{\text{thl}}^2 < \omega/k_z^2 < \nu_{\text{thi}}^2 < \nu_{\text{the}}^2 \) where \( \nu_{\text{thj}}^2 \equiv 2T_j/m_j \), \( j = i, e \) and \( I \). Moreover, we assume that \( T_I \simeq T_i \simeq T_e \) and the effects of the longitudinal thermal conductivity of the heavy population are relatively small. Under these conditions the relevant dispersion function is given by

\[
(\omega^2 - \omega_{\text{IA}}^2)(\omega(1 + i\epsilon_i) + \omega_{sI}) = (3/5) (\omega - \omega^*_{sI}) \omega_{\text{IA}}^2 \Delta,
\]

where \( \omega_{\text{IA}}^2 \equiv (5/3) (k_i^2 T_i/m_i) \), \( \omega_{sI} \equiv [Z k_i c T / (eBn)] (dn_I/dx) \), \( \nu / \Delta \equiv n_i/T_i + n_e/T_e \), \( \Delta \equiv Z^2 n_I T_i / (\nu n_I) \), \( \omega^*_{sI} \equiv \omega_{sI}/\Delta \) and \( \epsilon_i \equiv (m_i T_i/n_i) \left[ (c k_i^2 / eB) (d T_i/dx) / (k_i^2 D_{\text{eff}}') \right] \left[ 1 - \eta^* / \eta_i \right] \),

where \( D_{\text{eff}}' \) is the effective main ion parallel thermal diffusivity introduced to simulate wave-particle resonances, \( \eta_i \equiv d \ln T_i / d \ln n_i \) and \( \eta_i^* \) is the critical value of \( \eta_i \) above which the instability occurs. If we consider the case in which \( \Delta < 1 \) and \( \epsilon_i \) is small, Eq. (1) has solution

\[
\omega \simeq \omega_{\text{IA}} + 3\Delta (\omega_{\text{IA}} - \omega^*_{sI}) / 10,
\]

which corresponds to the mode with phase velocity in the di-
rection of the electron diamagnetic drift. This mode can be excited where there is a temperature "knee" but no density "knee" (relatively large \( \eta_i \) involved, a characteristic [6, 8] of the I-Regime) by the local main ion temperature gradient in conjunction with the finiteness of the impurity temperature for \( \omega'_I < \omega_{IA} \).

The directions of the particle flows produced by the considered mode can be estimated by the quasi-linear approximation. Taking conventional averages over y and z denoted by \( \langle \rangle \), we observe that, since \( \hat{n}_e \) and \( \hat{\phi} \) are in phase, \( \Gamma_e \equiv \langle \hat{n}_e \hat{\nabla}_E \rangle = 0 \) that is, there is no net electron flow. The quasi-neutrality implies that \( Z \langle \hat{n}_i \rangle \) remains unchanged, since \( \hat{\phi} \) and \( \hat{\nabla}_i \) are in phase, \( \langle \hat{n}_i \hat{\nabla}_E \rangle = -\langle \hat{n}_i \hat{\nabla}_E \rangle \). Then we argue that \( \varepsilon_s^i \) remains constant.

In particular, since \( \langle \hat{n}_i \hat{\nabla}_E \rangle \approx -n_i \langle \hat{T}_i \hat{\nabla}_E \rangle / T_i \) we obtain

\[
\langle \hat{n}_i \hat{\nabla}_E \rangle \approx \left( \frac{1}{\Gamma_e} \frac{d \Gamma_e}{d x} \right) \frac{n_i}{k_i^2 D_{\text{eff}}} \left( \langle \hat{\nabla}_E \rangle^2 \right) \left( 1 - \frac{\eta_i}{\eta_i} \right) .
\]

(2)

This shows that the main ion transport is inward for \( \eta_i > \eta_i^\text{f} \), while the impurity transport is outward, a feature consistent with the fact that the impurities are observed to be expelled toward the edge of the plasma column in the presence of the considered mode. Thus, we can argue that the transport of the impurity population raises the value of \( \eta_i \) to the saturated state, starting from the initial stage where \( \eta_i \approx -e \hat{\phi} n_i (1 + i \tilde{\varepsilon}_i) / T_i \) in the saturated state, starting from the initial stage where \( \hat{n}_i \approx -e \hat{\phi} n_i (1 + i \tilde{\varepsilon}_i) \) and \( \tilde{\varepsilon}_i \) is given by Eq. (8.2) in Ref. [2]. Then we argue that \( \varepsilon_i^s \) remains constant.

### Poloidal Magnetic Field Fluctuations

The electromagnetic fluctuations accompanying the density fluctuations produced by the mode are observed experimentally, i.e., the estimated poloidal magnetic field fluctuations are \( \tilde{B}_p \simeq (3 - 8) \times 10^{-4} \text{T} \) [11] while the estimated density fluctuations are \( \tilde{n}_e / n_e \approx 10^{-2} \times \alpha_n \) [12], where \( \alpha_n \approx 1 \). We note that

\[
\hat{A}_i \simeq \frac{4 \pi \hat{j}_i}{c k_i^2} \simeq -i 4 \pi \frac{\nabla \cdot \hat{J}_\perp}{c k_i^2} .
\]

(3)

In particular, \( \nabla \cdot \hat{J}_\perp = \nabla \cdot \langle (n_i + Z n_I - n_e) \hat{\nabla}_E \rangle = 0 \) at the stage where the linear description of the mode is valid. We can speculate that there is a nonvanishing \( \nabla \cdot \hat{J}_\perp \) at the saturated stage of the mode evolution in order to justify the observed electromagnetic fluctuations.

We argue that in the saturated state \( \hat{n}_e / n_e \) remains \( \approx e \hat{\phi} / T_e \) and the electron radial velocity is \( \hat{v}_E \). Thus \( \Gamma_{es} \equiv \langle \hat{n}_e \hat{\nabla}_E \rangle = 0 \). Consistently with the experimental observation that injected impurities are promptly expelled, the impurity radial flux is considered to become greatly reduced relative to that estimated by the quasi-linear theory that is valid at the start of the mode evolution. Consequently, \( |\Gamma_{ix}| \equiv \langle |\hat{n}_i \hat{\nabla}_E| \rangle \ll |\langle \hat{n}_i \hat{\nabla}_E \rangle| \). We assume that \( \hat{n}_i \) remains of the form \( \hat{n}_i \approx -e \hat{\phi} n_i (1 + i \tilde{\varepsilon}_i) / T_i \) in the saturated state, starting from the initial stage where \( \hat{n}_i \approx -e \hat{\phi} n_i (1 + i \tilde{\varepsilon}_i) \) and \( \tilde{\varepsilon}_i \) is given by Eq. (8.2) in Ref. [2]. Then we argue that \( \varepsilon_i^s \) remains constant.
close to $\tilde{\epsilon}_i$ and $\tilde{\phi}_x \simeq \tilde{\phi}_x + \Delta \tilde{\phi}_x$, where $\Delta \tilde{\phi}_x \simeq -i \varepsilon_s \tilde{\phi}_x$. Correspondingly, $\tilde{\epsilon}_x \simeq \tilde{\epsilon}_x + \Delta \tilde{\epsilon}_x$, and since $Z\tilde{n}_I \simeq \tilde{n}_e - \tilde{n}_i$ we have $\Delta \tilde{\phi}_x \simeq -i \varepsilon_s \tilde{\phi}_x \tilde{n}_I \tilde{T}_i \tilde{n}_i / (\tilde{n}_I \tilde{T}_i)$. Thus a current density

$$\tilde{J}_x \simeq e (n_i \Delta \tilde{\phi}_x + Zn_I \Delta \tilde{\phi}_x) = -i \varepsilon_s n_i \tilde{\epsilon}_x \left[ 1 + Zn_I \tilde{T}_i / (\tilde{n}_I \tilde{T}_i) \right] \simeq -i \varepsilon_s n_i \tilde{\epsilon}_x$$

(4)

has to be considered. We shall take $\nabla \cdot \tilde{J}_x \simeq \alpha_J \partial \tilde{J}_x / \partial x \simeq \alpha_J \tilde{J}_x / \Delta r$ and in view of Eqs (3), (4) and that $\tilde{A}_n \simeq \int B_\theta dr \simeq \tilde{B}_\theta r \Delta r$ we obtain, for $d_i^2 \equiv c^2 / \omega_{pi}^2$,

$$\tilde{B}_\theta \simeq i \alpha_J 4 \pi \varepsilon_s n_i \tilde{\epsilon}_x \left( k \Delta r k_i \Delta r B \right)^{-1} \left( \frac{c T_i}{k} \right) \frac{B}{e B d_i^2 \Omega_{ci}} \epsilon_i^2 \times \alpha_n \times \alpha_J \times 10^{-2}.$$  

(5)

For $k_0 \simeq l_0 / (q R_0) \simeq 2 / 35 \text{ cm}^{-1} \left( l_0 / 10 \right) \left( 2.5 / q \right) \left( 70 \text{ cm} / R_0 \right)$, $\Delta r \sim 1 \text{ cm}$, $k \sim 2 \text{ cm}^{-1}$, $T_i \sim 600 \text{ eV}$, $n_i \sim 10^{14} \text{ cm}^{-3}$ and $B \simeq 5 \times 10^4 \text{ G}$, referring to the relevant experiments, Eq. (5) reduces to

$$\left| \frac{\tilde{B}_\theta}{B} \right| \simeq 1 / 2 \times \epsilon_i^2 \times \alpha_n \times \alpha_J \times 10^{-4} \left[ 2 / \Delta r \right] \left[ 2 / 35 \right] \left[ c T_i / k e B \right] \left( 1.2 \times 10^6 \text{ cm}^2 / \text{ sec} \right)$$

$$\cdot \left[ \frac{3 \text{ cm}}{d_i} \right] \left[ \frac{2.4 \times 10^8 \text{ rad} / \text{ sec}}{\Omega_{ci}} \right]$$

and we can see that values of $\tilde{B}_\theta$ are in the range that has been estimated from experimental observations.

**Spontaneous Rotation**

The spontaneous rotation observed in the I-Regime is in the direction of the main ion diamagnetic velocity (co-current) and, following the "accretion theory" [10], a process to scatter angular momentum to the wall generating a recoil in the opposite direction, that of the ion diamagnetic velocity, and an "inflow" process to transport the generated angular momentum from the edge of the plasma column toward the center, should be present.

The heavy particle mode that is excited at the edge of the plasma column can provide the scattering of angular momentum in the same direction as that of electron diamagnetic velocity.

**Conclusions**

We have identified a new mode that can be excited in a multi-component plasma with a massive particle population. This has: i) a phase velocity in the direction of the electron diamagnetic velocity, ii) produces an outward transport of the heavy particle (impurity) population and iii) involves the light ion population temperature gradient and the effective longitudinal thermal conductivity as the driving factors. These characteristics and the fact that the mode is shown to produce measurable electromagnetic fluctuations make it a suitable candidate to explain the features of the 200 kHz mode found experimentally in the I-confinement Regime where the
plasma thermal energy is well confined while the impurities are segregated at the edge of the plasma column.

References


