Expected accuracy of fuel ion ratio measurements by collective Thomson scattering at TEXTOR

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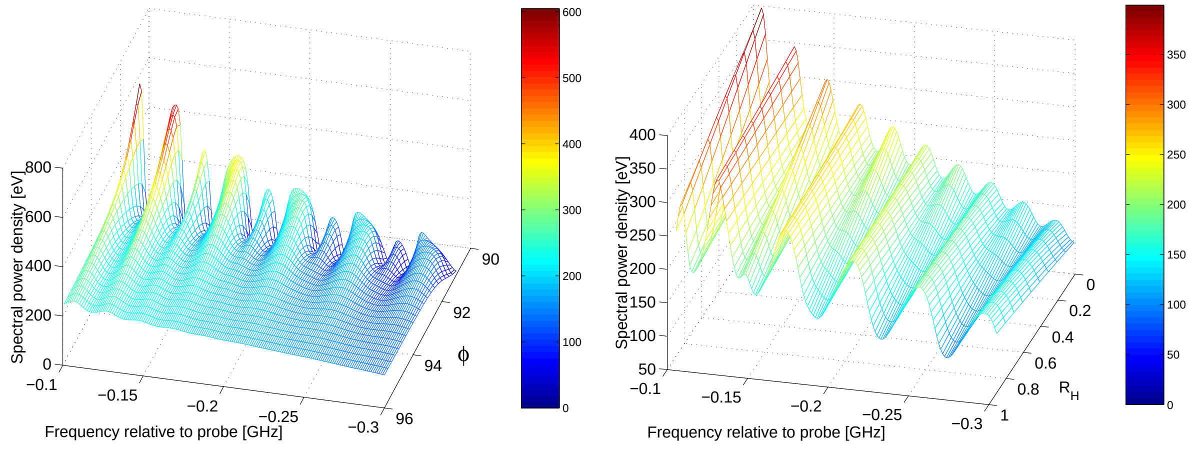
Introduction

In magnetically confined fusion experiments and for future reactors, measurements of the fuel ion ratio – the ratio between fuel ion densities – are of intrinsic scientific interest and will further be important for plasma control and machine protection. The techniques currently used for fuel ion ratio measurements – most notably neutral particle analysis – may not be applicable in the core of a fusion reactor [1]. It is therefore important to develop alternatives. Microwave based collective Thomson scattering (CTS) diagnostics are well suited for reactor environments and provide access to the dynamics of confined ions by measuring the spectrum of probe radiation scattered by plasma fluctuations excited by ion motion. CTS measurements thus yield information on the ion velocity distribution and on certain plasma waves. For specific scattering geometries, CTS spectra contain signatures of ion Bernstein waves (IBWs) which are highly sensitive to the ion species mix. Measurements of IBW signatures in CTS spectra have therefore been suggested as a new diagnostic principle for measurements of the fuel ion ratio. Previous feasibility studies have found that such a diagnostic could fulfill the measurement requirements for ITER [2], and that it could be integrated in the CTS system foreseen to measure fast ion velocity distributions on ITER [3]. The next natural step is to perform proof-of-principle experiments in current machines demonstrating the ability to measure IBW signatures in CTS spectra, their sensitivity to plasma composition, and the ability to infer the fuel ion ratio – or in current machines the equivalent hydrogen to deuterium density ratio $R_{\mathrm{H}} = n_{\mathrm{H}}/(n_{\mathrm{D}} + n_{\mathrm{H}})$. The CTS receiver at TEXTOR was recently modified for such experiments, which require higher frequency resolution than was possible with the existing system [4]. Here we discuss the theoretical sensitivity to R_{H} of spectra measured with the modified receiver, and the expected accuracy with which R_{H} could be inferred from such spectra. Results of experiments to measure IBW signatures will be reported elsewhere.

IBW signatures in CTS spectra

In CTS experiments, probe radiation (the CTS system at TEXTOR uses microwave gyrotron radiation at 110 GHz) with frequency and wave vector $(\nu^{\mathrm{i}}, \mathbf{k}^{\mathrm{i}})$ scatters off plasma fluctuations, and scattered radiation $(\nu^{\mathrm{s}}, \mathbf{k}^{\mathrm{s}})$ is picked up by a receiving antenna. The probe launching an-



(a) Variation with ϕ

(b) Variation with R_{H}

Figure 1: Theoretical spectra calculated with parameters relevant for TEXTOR: $n_e = 3 \times 10^{19}$ m^{-3}, $T_e = 1.1$ keV, $T_i = 1.1$ keV, $B_t = 2.6$ T. The spectrum is shown for frequencies from 300 MHZ to 100 MHz below the probe frequency corresponding to the range monitored with the modified CTS receiver. In (a) R_{H}=0.5 while ϕ is varied from 91° to 95°. In (b) $\phi = 93°$ while R_{H} is varied from 0.01 to 0.99.

tenna and the receiving antenna together determine the scattering geometry and the orientation of $\mathbf{k}^{\mathrm{s}}$. The measurement resolves fluctuations in the plasma with frequency and wave vector $(\nu^{\delta}, \mathbf{k}^{\delta}) = (\nu^{\mathrm{s}} - \nu^{\mathrm{i}}, \mathbf{k}^{\mathrm{s}} - \mathbf{k}^{\mathrm{i}})$. When the length scale of the resolved fluctuations is greater than the Debye length, $\mathbf{k}^{\delta}\lambda_D < 1$, the spectrum is sensitive to collective fluctuations in the plasma driven by ion motion. ν^{δ} is essentially proportional to the ion velocity in the direction of $\mathbf{k}^{\delta}$ so CTS measurements are sensitive to one-dimensional projections of the ion velocity distribution along $\mathbf{k}^{\delta}$.

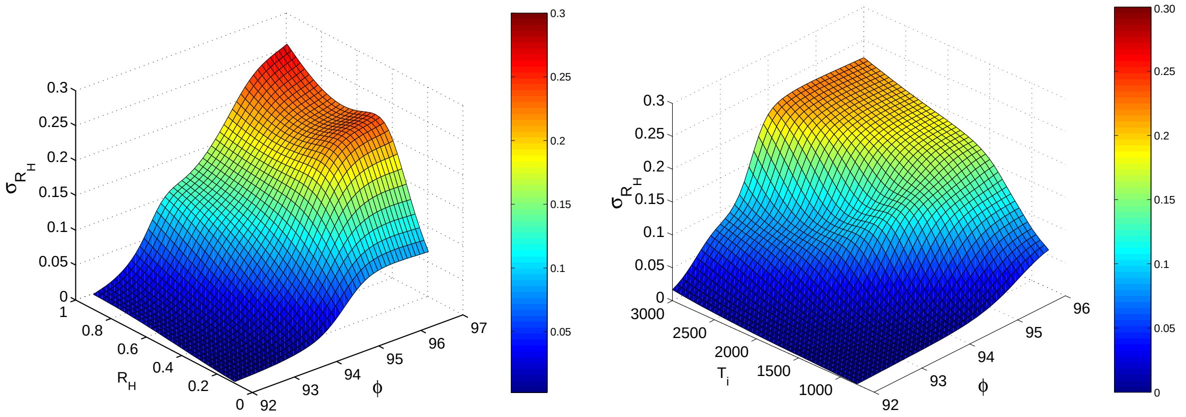
IBWs are electrostatic solutions to the hot plasma dispersion relation and propagate at frequencies between the ion cyclotron harmonics of each ion species in the plasma. IBWs are weakly damped when propagating in directions close to perpendicular to the magnetic field. They are therefore readily excited by thermal ion motion and will, under usual circumstances, be intrinsically present in tokamak plasmas with no external drive. In directions away from perpendicular the damping increases strongly, and it entirely suppresses the waves in directions far from perpendicular. The CTS spectrum therefore only contains IBW signatures when $\phi = \angle(\mathbf{k}^{\delta}, \mathbf{B}) \sim 90°$. Figure 1(a) shows theoretical CTS spectra with IBW signatures. The spectra were calculated with a kinetic and fully electromagnetic model of CTS [5] assuming plasma parameters and frequencies ranges relevant to the TEXTOR CTS system. The IBWs give rise to peaks in the CTS spectrum at intervals corresponding to the cyclotron frequencies of the dominant ion species – here hydrogen with $\nu_{\mathrm{c,H}} = 40$ MHz and deuterium with $\nu_{\mathrm{c,D}} = 20$ MHz. The peak amplitude is high for $\phi \simeq 90°$ but decreases rapidly for angles further from perpendicular.

The relative heights of peaks related to different ion species depends on the relative densities of those ions. This is illustrated in Fig. 1(b) which shows spectra calculated for different values of R_{H}. At low R_{H} the spectrum displays peaks at intervals of 20 MHz corresponding to $\nu_{\mathrm{c,D}}$. At higher values of R_{H} IBWs related to hydrogen contributes to every second peak. These peaks largely maintain their amplitude while peaks with contributions only from IBWs related to deuterium gradually lose amplitude and disappear. Furthermore, radiation scattered by light ions, on average, has a higher Doppler shift due to the higher thermal velocity of those ions. Hydrogen and IBWs related to hydrogen therefore contribute more to the spectrum at frequencies far from the probe than deuterium. IBWs related to ions with different masses will also be damped differently since the damping is a finite Larmor radius effect. This means that the spectrum is sensitive independently to both the charge and the mass of the ions. Therefore it is in principle possible to distinguish ions with the same charge to mass ratio (and hence the same cyclotron frequency) but with different masses. These are the properties which give CTS measurements of IBW signatures the potential to become a diagnostic tool for the fuel ion ratio.

Expected accuracy of fuel ion ratio measurements

Besides the fuel ion ratio and scattering geometry, the CTS spectrum depends in a non-trivial way on a number of parameters including bulk electron- and ion temperatures, electron density, magnetic field strength, impurity densities, and the velocity distributions of any non-thermal ion populations. To infer the fuel ion ratio, the spectrum must be fitted with a forward model for CTS. Here we use a Bayesian least squares method of inference [6] previously used to infer fast ion velocity distributions [7]. This allows the fit to be optimized taking into account any prior knowledge about all relevant parameters and their uncertainties. In this approach, the probability distributions describing the measured spectrum, the prior information about model parameters, and the resulting posterior distribution (the state of knowledge after the measurement) are all assumed to be multivariate normal distributions. After the measurement, the uncertainty on a given parameter is related to the width of the posterior distribution and it may be expressed as a function of the uncertainties in the prior information, the uncertainties in the measured spectrum, and the Jacobian for the best fit spectrum. This means that for given assumptions about the uncertainties in the prior information and the measured spectrum, the uncertainty of the inferred fuel ion ratio can be estimated theoretically.

Figures 2(a) and 2(b) show examples of theoretically expected posterior uncertainties for R_{H} for realistic ranges of R_{H}, ϕ and T_{i} on TEXTOR. The model parameters kept fixed are identical to those used in Figs. 1(a) and 1(b). Their uncertainties are given in the figure caption. The uncertainties in the measured spectra are assumed to be 2% of the signal strength – as was found in initial experiments with the modified receiver and as is expected from theory [4]. The prior uncertainty in R_{H} was 0.5, and the posterior uncertainty is significantly reduced from



(a) σ_{R_H} for varying ϕ and R_H

(b) σ_{R_H} for varying ϕ and T_i

Figure 2: Theoretically calculated posterior uncertainties for model parameters as in Fig. 1 and prior uncertainties: $\sigma_{n_\mathrm{e}} = 3\times10^{18}\ \mathrm{m}^{-3}$, $\sigma_{T_\mathrm{e}} = 0.4$ keV, $\sigma_{T_\mathrm{i}} = 0.4$ keV, $\sigma_{B_t} = 0.1$ T, $\sigma_\phi = 3^\circ$, $\sigma_{R_\mathrm{H}} = 0.5$. In (*a*) R_H and ϕ are varied while other parameters are kept constant. In (*b*) $R_\mathrm{H} = 0.5$ and ϕ and and T_i are varied while other parameters are kept constant. σ_{R_H} is significantly reduced from its prior value when ϕ is close to 90°.

this value when ϕ is close to 90° – i.e. when the spectrum contains IBW signatures. In the frequency range monitored by the CTS receiver at TEXTOR, the signatures related to hydrogen tends to dominate over those related to deuterium due to the larger Doppler shift. At high R_H the spectum is therefore less sensitive to changes in R_H and the posterior uncertainty is higher. The IBWs can be damped at high ion temperatures causing the IBW signatures to be weaker, which can also lead to higher posterior uncertainties in R_H. Nevertheless, the calculations show that, within broad parameter ranges, the diagnostic can be expected to measure R_H with an uncertainty around or below 0.1 at TEXTOR. Initial studies indicate that similar accuracies can be expected for ITER standard scenarios [2]. This will be investigated in detail elsewhere.

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