Discussion on Slow Light in Magnetized Plasma by Electromagnetically Induced Transparency

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ABSTRACT This paper discusses an analogous state to “dark-state polariton” [M. Fleischhauer and M. D. Lukin, Phys. Rev. Lett. 84, 5094 (2000)] in magnetized plasma under electromagnetically induced transparency (EIT). Propagation of a coupled state of a probe-wave electric field and longitudinal plasma oscillation can be stopped as its waveform is maintained if a pump-wave is properly controlled in time. The “stop-light” is achieved by adiabatic transition between states in which the probe-wave field is dominant and the longitudinal plasma oscillation is dominant.

Conventional EIT is a phenomenon in which optical properties of a material are drastically changed by quantum interference between eigenstates of matter when irradiated by two lights\(^1\). The EIT is the basis of several applications, such as slow light, information transfer between matter and light and so on. A theoretical idea by Harris\(^2\) initiated research of a new EIT assuming plasma as a medium, and subsequently possibility of EIT in magnetized plasma was shown by researchers\(^3\) by theoretical/numerical analyses. Those theories predict that injection of pumping wave alters susceptibility of plasma to right-hand circular polarized (RHCP) electron cyclotron wave. As a result, the RHCP wave no longer resonates with electrons at the electron cyclotron frequency, being transmitted. The EIT in plasmas does not invoke quantum mechanics while the conventional EIT requires quantum mechanics. Instead, the EIT in plasmas is realized by collective excitation of a medium.

Under the EIT condition that two electromagnetic waves of probe and pump are injected into a slowly varying (in space and time) magnetized plasma, the probe-wave frequency
component of induced current density \( J(r,t) \) by the wave field can be expressed as linear response to the probe wave electric field \( E_1 \) including the pump field \( E_0 \) as a parameter

\[
J(r,t) = \left( \vec{\sigma}^T_{EIT} + \vec{k}_{EIT}^T \right) \cdot a_i(r,t)e^{i \theta_{EIT}}.
\]

(1)

Here, \( E_1(r,t) = a_i(r,t)e^{i \theta_{EIT}} \), \( k(r,t) = \nabla \psi(r,t) \), \( \omega(r,t) = -\partial \psi(r,t)/\partial t \), where \( \psi(r,t) \) is rapidly varying over the scale length of a wave and period of the wave. \( a_i(r,t) \) slowly varies over scale length of the plasma inhomogeneity. See Ref. 6 for definition of other variables.

Substitution of eq. (1) to a wave equation \( \nabla \times \nabla \times E + \vec{k}^2 E/\omega^2 + \mu_0 \partial J/\partial t = 0 \) under the assumption that \( k = \nabla \psi \) and \( \omega = -\partial \psi/\partial t \) are zero order quantities, \( a_i, \nabla \) and \( \partial/\partial t \) are first order gives a first order equation \( \vec{\varepsilon} \cdot a_i = 0 \), \( \vec{\varepsilon} = c^2 \left( \kappa k - k^2 \vec{T} \right)/\omega^2 + \vec{1} + \vec{O}_{EIT}/\omega \) which provides the dispersion relation. The second order equation for one-dimension (\( k = k_z \) and \( \nabla = (0,0,\partial/\partial z) \)) is,

\[
\frac{\partial}{\partial t} a_\perp + \frac{c^2 k_z}{\omega} \frac{\partial}{\partial z} a_\perp + \frac{a_\perp}{2\omega} \frac{\partial}{\partial t} \omega + \frac{c^2 a_\perp}{2\omega} \frac{\partial}{\partial z} k_z = -\frac{1}{2\omega} \frac{\partial}{\partial t} \left\{ \left( Q_{xx} + iQ_{xy} \right) a_\perp \right\} - \frac{1}{2\varepsilon_0} \left\{ \left( \kappa_{xx} + i\kappa_{xy} \right) a_\perp \right\},
\]

(2)

if \( Q_{xx} = Q_{yy} \), \( Q_{xy} = -Q_{yx} \) and \( Q_{zz} = Q_{zz} = 0 \). Here, \( a_\perp \equiv a_{ix} - ia_{iy} \), \( Q_{ij} \) and \( \kappa_{ij} \) are \( i,j \) component of \( \vec{O}_{EIT} = i\vec{k}_{EIT}^T/\varepsilon_0 \) and \( \vec{k}_{EIT}^T \), respectively. This equation describes temporal evolution of right hand circular (RHC) component (\( a_{ix} - ia_{iy} \)) of amplitude of the probe wave. If time scale of frequency change of the probe wave is much slower than that of wave envelope change and spatial variation of wavelength of the probe wave is much smaller than the envelope change, condition \( \delta t_{\text{probe}} << \delta t_{\text{pump}} \) makes \( \left( \kappa_{xx} + i\kappa_{xy} \right) a_\perp \) in the right hand side (RHS) of eq. (2) simple.

\[
\frac{\partial}{\partial t} a_\perp + \frac{c^2 k_z}{\omega} \frac{\partial}{\partial z} a_\perp = -\frac{1}{2\omega} \frac{\partial}{\partial t} \left\{ \left( Q_{xx} + iQ_{xy} \right) a_\perp \right\} - \frac{i}{2\varepsilon_0} \left\{ \left( \partial(\sigma_{xx} + i\sigma_{xy})/\partial \omega \right) \frac{\partial a_\perp}{\partial t} \right\}.
\]

(3)

Here, \( \delta t_{\text{pump}} \) and \( \delta t_{\text{probe}} \) are the pulse width of the pump and the probe waves, respectively.

The second term of the RHS is RHC component of current which stems from frequency response of susceptibility of the medium. The first term of the RHS is RHC component of current induced by RHCP component of probe-wave electric field. Replacement

\[-i(\sigma_{xx} + i\sigma_{xy})/(\varepsilon_0 \omega) = -(Q_{xx} + iQ_{xy})/\omega = \chi_{EIT} \]

can be done in these terms, where the
susceptibility for EIT regime $\chi_{\text{EIT}}$ is given by Hur, et al. $^5$

$$\chi_{\text{EIT}} = \frac{\omega_t^2}{\omega} \delta \Omega + \delta \Omega_0(k_0) \Omega_t^2 - \delta \Omega^2$$

and $a_{1t} = e a_{1t} / (m c \omega)$, where $\delta \Omega = \omega - \Omega_{ce}$, $\Omega_t = e k_0 |a_0| \sqrt{\Omega_{ce}}$. \(\text{(4)}\)

Here, $\Omega_t$, $a_0$, $k_0$ and $m$ are Rabi frequency, normalized amplitude and wave number of the pump-wave electric field and electron mass, respectively (see ref. $^5$ for definition of other variables.). When $a_0$ is strong enough, $\chi_{\text{imag}} \to 0$ indicating disappearance of resonant absorption of the probe-wave at $\omega = \Omega_{ce}$. The EIT window emerges and expands as $a_0$ increases. Moreover even if $a_0$ decreases to some extent (but $\neq 0$), $\chi_{\text{imag}}$ can be small enough and $\partial \chi_{\text{real}} / \partial \omega$ is large, indicating slow light propagation without resonant absorption.

When $\chi_{\text{EIT}} = \chi_{\text{real}} + i \chi_{\text{imag}} \sim 0$ such as in case of large $a_0$, the first term of the RHS of eq. (3) vanishes. The wave equation of envelop of probe-wave can be written in the form:

$$\frac{\partial a_t}{\partial t} + v_s \frac{\partial a_t}{\partial z} = 0, \quad v_s = c^2 k_z / \omega_{\text{probe}} / \left(1 + \frac{1}{2} \omega_{\text{probe}} \frac{\partial \chi_{\text{EIT}}}{\partial \omega_{\text{probe}}} \right). \quad \text{(5)}$$

When $v_s$ is a function of only $z$, achievable lowest group velocity is limited as follows $^4$ $^7$. For EIT medium, $\chi_{\text{EIT}}$ changes sharply within the EIT window, whose width is $w$. Slope of $\chi_{\text{EIT}}$ is estimated as $\partial \chi_{\text{EIT}} / \partial \omega_{\text{probe}} \sim \Delta \chi_{\text{EIT}} / w$ $\sim \Delta \chi_{\text{EIT}} / \Omega_t$, where $\Delta \chi_{\text{EIT}}$ is the variation of $\chi_{\text{EIT}}$ in the window ($\Omega_{ce} \pm \Omega_t$). Note that Rabi frequency $\Omega_t$ is proportional to the pump intensity $a_0$.

$$v_s \sim (c^2 k_z / \Omega_{ce}) / (\Delta \chi_{\text{EIT}} / w) \sim (c^2 k_z / \Delta \chi_{\text{EIT}} \Omega_{ce}) 2 \Omega_t \sim (c^2 k_z / \Delta \chi_{\text{EIT}} \Omega_{ce}) a_0. \quad \text{(6)}$$

Spectrum of the probe-pulse (envelop) must be contained within the EIT window $w$. Namely, the spectral width of the envelope must satisfy the following condition.

$$1 / \delta t_{\text{probe}} < w \sim 2 \Omega_t \sim a_0. \quad \text{(7)}$$

That is, the lowest achievable group velocity is limited by pulse width of the envelope $\delta t_{\text{probe}}$ when $v_s$ is a function of $z$ and constant in $t$. However, if $v_s$ is a function of $t$, the solution has a form of $a_1(z,t) = h(z - \int v_s(t') dt')$, $v_s = c^2 k_z / \omega_{\text{probe}} / \left(1 + \omega_{\text{probe}} \partial \chi_{\text{EIT}} / \partial \omega_{\text{probe}} \right)^{1/2}$, indicating envelop of the probe-wave propagates in the $z$-direction and spatial profile of the wave packet is maintained at any $t$. Deceleration of the wave propagation in this scheme is realized by compression in time in contrast to that by the preceding scheme due to compression in space.
This method prevents the deceleration of light from being restricted by the effective band width of the medium indicated by eqs. (6) and (7) because initial band width of the probe-wave pulse is kept constant even during the wave packet is slowed down.

If $1/\delta t_{\text{probe}} \ll \omega_p, \omega_l$ and \[ \left| v_{+} \right| = \left| \omega_0 e_0 \right| / \left| m_e \omega_0 \right| \ll \left| \omega_0 e_0 \right| / \left| m_e \omega_0 \right| \], current induced by the probe-wave $a_i^k(t)$ is written as $j_i = -en_0 v_{+} = 2(m_e \omega_0/(ek_0)) \left\{ e_0 \omega_p^2 / \left\{ \omega_0^2 \right\} \right\} \partial_{\omega_0} a_i^k(t)$ . This indicates $\chi_{\text{eff}} = \chi_{\text{real}} + i\chi_{\text{imag}} \approx 0$ and $2(m_e \omega_0/(ek_0)) \left\{ e_0 \omega_p^2 / \left\{ \omega_0^2 \right\} \right\} \approx \omega_0 \left( \partial \chi_{\text{eff}} / \partial \omega_0 \right) \propto v_g^{-1}$ . Also the following relation is derived from a simplified equation of motion of electrons,

$$ k_\parallel a_i^k(t)/\omega_0 = 2a_i^k(t) \left\{ -i\omega_p \tilde{\xi} \right\} . $$(8)

This indicates that ratio between the probe field $2a_i^k(t)$ and the longitudinal oscillation $-i\omega_p \tilde{\xi}$ is determined by the pump electric field $a_i^k(t)$ . The coupled state of $2a_i^k(t)$ and $-i\omega_p \tilde{\xi}$ , which is regarded as a coupled state of light and matter in quantum field, propagates in the direction of $z$ at $v_g$ which is controlled by $a_i^k(t)$ as well. When “stop-light” is accomplished by adiabatic change of $2a_i^k(t)$ and $-i\omega_p \tilde{\xi}$ , the plasma oscillation dominates the coupled state. Energy of the probe wave field is stored into the plasma oscillation in the coupled state during the probe-wave is stopped. This is an analogous phenomenon to dark-state polariton in quantum system.4 Adiabatic control of the ratio between the probe field and the longitudinal oscillation through the pump-wave intensity realizes the manipulation of the group velocity of the probe-wave moreover, freeze of the wave propagation.

REFERENCES