Ferritic structures behind the first wall and RWM stability in tokamaks

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1. Introduction. Recently [1] the effect of ferromagnetic structures on the Resistive Wall Mode (RWM) growth rates have been studied analytically and verified numerically for the JET tokamak using the CREATE_L code [2]. It was found, consistent with previous results analytically obtained for somewhat different arrangements [3], that with ferromagnetic materials the growth rate γ is always greater than that without them. In the case considered [1] the growth rate increase $\Delta\gamma$ did not depend on the plasma configuration, but only on the poloidal mode number and on the parameters of the ferromagnetic structures behind the first wall. Independence of $\Delta\gamma$ on plasma pressure and current would substantially simplify the feedback plasma stabilization in ITER where the presence of ferromagnetic steel is envisaged in test blanket modules [4]. Acknowledging the importance of this result in both academic and practical respects, we give here a derivation of the dispersion relation, alternative to that in [1].

2. Model. Analytical model is based on cylindrical approximation, which allows the mode separation. We consider the plasma surrounded by two walls, with the wall facing the plasma as simply resistive and thin both geometrically and magnetically, while the second nonconductive wall of larger radius is treated as thick, ferromagnetic, but nonconductive. The key element of our approach is incorporation of the magnetic permeability μ into the boundary conditions

$$\langle \mathbf{n}\cdot\mathbf{b}\rangle = 0 \text{ and } \langle \mathbf{n}\times\mathbf{b}/\mu\rangle = 0, \tag{1}$$

which must be satisfied (with different μ) at four wall-vacuum interfaces. Here $\mathbf{b}$ is the magnetic perturbation, the brackets $\langle ...\rangle$ mean the jump across the surface. In the vacuum and in the first wall $\mu = \mu_0 = 4\pi\times10^{-7}$ H m^{-1}.

Outside the plasma, except for the first wall, the perturbation $\mathbf{b}$ satisfies the equations

$$\nabla\cdot\mathbf{b} = 0, \qquad \nabla\times(\mathbf{b}/\mu) = 0 \tag{2}$$

and decreases to zero at infinity. Equations (2) are complemented by the equation for $\mathbf{b}$ in the first wall, which is, for constant conductivity σ and $\hat{\mu} \equiv \mu/\mu_0 = 1$,

$$\mu_0\sigma\partial\mathbf{b}/\partial t = \nabla^2\mathbf{b}. \tag{3}$$

We solve it in the thin-shell approximation. Finally, we have to match the outer solution for **b** to the inner one (in the plasma) at the plasma boundary. This gives the dispersion relation.

3. Solution. First, we solve (2) and (3) in the space behind the first wall. With $\mathbf{b} = \nabla\psi \times \mathbf{e}_z$ and

$$\psi = \sum \psi_m(r,t)\exp(im\theta - in\zeta), \tag{4}$$

where (r,θ,z) are the cylindrical coordinates and $\zeta = 2\pi z/L$ mimics the toroidal angle, we have from (2) (assuming $nr/(mR) << 1$ with positive m and n):

$$\psi_m = gr^m + hr^{-m}. \tag{5}$$

The time-dependent constants (amplitudes) g and h are different pairs in different regions. For each mode they have to be found by matching the solutions under the boundary conditions

$$\langle \psi_m \rangle = 0 \quad \text{and} \quad \left\langle \frac{1}{\mu}\frac{d\psi_m}{dr} \right\rangle = 0 \tag{6}$$

at the wall surfaces, as implied by (1). Let us proceed starting from the outmost surface.

<u>The vacuum region outside the second wall</u> ($r > r_{w2} + d_{w2}$). With $g = 0$ there we have

$$\frac{r\psi_m'(r)}{\psi_m} = -m, \tag{7}$$

up to the outer surface of this wall.

<u>The region inside the second wall</u> ($r_{w2} < r < r_{w2} + d_{w2}$). Here (5) is valid because we assume this wall nonconducting and $\hat{\mu} = const$. We can put it in the form

$$\psi_m = Ay^m + Cy^{-m} \tag{8}$$

with $y = r/(r_{w2} + d_{w2})$. Then

$$\frac{r\psi_m'(r)}{\psi_m} = \frac{y\psi_m'(y)}{\psi_m} = m\frac{Ay^m - Cy^{-m}}{Ay^m + Cy^{-m}} = m\frac{\alpha y^{2m} - 1}{\alpha y^{2m} + 1}, \tag{9}$$

where $\alpha \equiv A/C$. According to (6) and (7), at the outer side of the wall ($y = 1$) this must be

$$\frac{r\psi_m'(r)}{\psi_m} = m\frac{\alpha - 1}{\alpha + 1} = -m\hat{\mu}. \tag{10}$$

This gives us

$$\alpha = -\frac{\hat{\mu} - 1}{\hat{\mu} + 1}. \tag{11}$$

At the inner side of the second wall, where $y = r_{w2}/(r_{w2} + d_{w2})$, equation (9) yields

$$\frac{r\psi_m'(r)}{\psi_m} = m\frac{\alpha\varepsilon_w - 1}{\alpha\varepsilon_w + 1}, \tag{12}$$

where

$$\varepsilon_w \equiv \left(\frac{r_{w2}}{r_{w2} + d_{w2}} \right)^{2m} . \tag{13}$$

With α given by (11) the relation (12) can be rewritten as

$$\frac{r\psi_m'(r)}{\psi_m} = -\hat{\mu}(m + \Gamma_m^c), \tag{14}$$

where

$$\Gamma_m^c \equiv -m \frac{\hat{\mu}^2 - 1}{\hat{\mu}} \frac{1 - \varepsilon_w}{\hat{\mu} + 1 - \varepsilon_w(\hat{\mu} - 1)} . \tag{15}$$

This is Γ_m^c introduced by Eq. (53) in [3]. Note that in order to incorporate the permeability effects we have to consider the second wall with proper account of its finite thickness [3].

The vacuum region between the two walls ($r_{w1} < r < r_{w2}$). First, according to (6) and (14), we have the next boundary condition for the vacuum solution at $r = r_{w2}$:

$$\frac{r\psi_m'(r)}{\psi_m} = -(m + \Gamma_m^c) . \tag{16}$$

Calculations for this region are similar to those for the second wall, but $\hat{\mu} = 1$ now. Here, for convenience of derivations, we present (5) in the form

$$\psi_m = Eu^m + Du^{-m}, \tag{17}$$

where $u \equiv r / r_{w2}$. Then, similar to (9),

$$\frac{r\psi_m'(r)}{\psi_m} = \frac{u\psi_m'(u)}{\psi_m} = m \frac{\kappa u^{2m} - 1}{\kappa u^{2m} + 1} \tag{18}$$

with $\kappa \equiv E / D$. Equating this to (16) at $u = 1$ (at the inner side of the second wall), we obtain

$$\frac{r\psi_m'(r)}{\psi_m} = m \frac{\kappa - 1}{\kappa + 1} = -(m + \Gamma_m^c), \tag{19}$$

which gives us, with Γ_m^c defined by (15),

$$\kappa = -\frac{\Gamma_m^c}{2m + \Gamma_m^c} . \tag{20}$$

At the position of the first wall ($u = r_{w1} / r_{w2}$) we have from (18):

$$\frac{r\psi_m'(r)}{\psi_m} = m \frac{\kappa - x_2^{2m}}{\kappa + x_2^{2m}}, \tag{21}$$

where $x_2 \equiv r_{w2} / r_{w1}$. This is equivalent to

$$\frac{r\psi'_m(r)}{\psi_m} = -m + \Delta\Gamma_m^f \tag{22}$$

at $u = r_{w1}/r_{w2}$, where

$$\Delta\Gamma_m^f \equiv \frac{2m\kappa}{\kappa + x_2^{2m}} \tag{23}$$

or, with account of (20),

$$\Delta\Gamma_m^f = -2m\frac{\Gamma_m^c}{\Gamma_m^c(x_2^{2m}-1)+2mx_2^{2m}} \,. \tag{24}$$

Equation for the first wall. According to [5], where a geometrically similar (but different in terms of $\hat{\mu}$ and σ) configuration with two walls has been analysed, at the outer side of the first wall we have, assuming $\psi_m \propto \exp(\gamma t)$,

$$\frac{r\psi'_m(r)}{\psi_m} = -(m + \Gamma_m^{w1}) + \gamma\tau_w \,, \tag{25}$$

where τ_w is the magnetic field diffusion time calculated in the thin-wall approximation [3, 5]. This is a result of the matching the outer solution for $\mathbf{b}$ to the inner one at the plasma boundary, and Γ_m^{w1} is a quantity determined by $r\psi'_m(r)/\psi_m$ in the plasma, as explained in [3]. This Γ_m^{w1} is a parameter representing the plasma in the problem.

By definition, the quantity (25) must the same as given by (22) obtained by solving the problem "from the other side". Equating them we come to the dispersion relation

$$\gamma\tau_w = \Gamma_m^{w1} + \Delta\Gamma_m^f \,, \tag{26}$$

which is an independent and complete proof of the main analytical result in [1]. This shows that at $\hat{\mu} \neq 1$ the growth rate increases by $\Delta\Gamma_m^f$ which does not depend on the plasma configuration, but only on the poloidal mode number and on properties of the ferromagnetic shell. A realistic example is presented in [1].

Acknowledgements. This work was supported in parts by Italian MIUR under PRIN grant 2008E7J7A3, by Consorzio CREATE, the Rosatom State Corporation (contract N.4a.52.03.10.1001 of 11.01.2010) and the Dutch-Russian Centre of Excellence on Fusion Physics and Technology (grant NWO-RFBR 047.018.002).

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