Runaway Electron Dynamics in High Atomic Number Plasmas

J.R. Martín-Solís 1, I. Fernández-Gómez 1, R. Sánchez 2

1 Universidad Carlos III de Madrid, Avda. de la Universidad 30, 28911-Madrid, Spain.
2 Fusion Energy Division, Oak Ridge National Laboratory, Oak Ridge, TN 37831, U.S.A.

1. Introduction
Disruption generated runaway electrons pose a significant danger for the operation of next-step devices like ITER, where as much as two thirds of the pre-disruption current might turn into runaway current. The development of efficient methods capable to mitigate the runaway damage during disruptions constitutes a challenging issue for a reactor-scale device. One of the most promising candidates is the massive gas injection (MGI) of high-Z impurities (mainly noble gases) for a fast plasma shutdown by means of a radiative collapse [1].

In this paper, we analyze the dynamics of runaway electrons in plasmas with high impurity content, which should be considered for a proper interpretation of the runaway behavior during disruptions, when a large impurity influx is expected following the thermal quench and, in particular, during MGI mitigation experiments. In the cold plasma following the disruption thermal quench, the impurity atoms are weakly ionized and the effect of the collisions with free and bound electrons, as well as the scattering by the full nuclear and the electron-shielded ion charge (depending on the electron impact parameter) should be taken into account. Here, after setting the proper collision terms to account for the collisions with partially stripped impurity atoms, the conditions for runaway generation and the possibility of achieving runaway suppression by MGI of high-Z impurities during disruptions will be investigated.

2. Collision terms
The friction force on a relativistic electron due to the collisions with the plasma electrons and ion species is given by:

\[
\vec{F}_{c\text{oll},e} = -\frac{e^4 m_e n_e \ln \Lambda_{ee} \gamma (\gamma + 1)}{4\pi \varepsilon_0} \frac{\gamma}{p^3} \vec{p}, \quad \vec{F}_{c\text{oll},i} = -\sum_j e^4 m_e Z_{\text{eff},j}^2 n_j n_e \ln \Lambda_{ej} \gamma \frac{\gamma}{p^3} \vec{p}
\]  

(1)

The dynamics of runaway electrons in tokamak plasmas has been traditionally analyzed taking only into account the collisions with the free plasma electrons, plasma protons and the scattering by the electron-shield charge, \( Z_{\text{av},j} \), of the impurity ions. The resulting friction force is:

\[
\vec{F}_{c\text{oll},e} = -\frac{e^4 m_e n_{e\text{f}} \ln \Lambda \gamma (\gamma + 1)}{4\pi \varepsilon_0^2} \frac{\gamma}{p^3} \vec{p}, \quad \vec{F}_{c\text{oll},i} = -\frac{e^4 m_e n_{e\text{f}} Z_{\text{eff}} \ln \Lambda \gamma}{4\pi \varepsilon_0^2} \frac{\gamma}{p^3} \vec{p}
\]  

(2)

where \( n_{e\text{f}} \) is the density of free electrons, \( Z_{\text{eff}} = \sum_j n_j Z_{\text{av},j}^2 n_{e\text{f}} \) is the effective ion charge and the same Coulomb logarithm, \( \ln \Lambda \), is assumed for all the plasma species.

However, in a plasma with high concentrations of partially stripped impurity atoms (as those typically found during disruptions), the collisions with the free-plasma electrons...
and the bound electrons in the impurity ions ($\vec{F}_{\text{coll},f}$ and $\vec{F}_{\text{coll},b}$, respectively) and the scattering by the plasma protons ($\vec{F}_{\text{coll},H}$), the electron-bound-shield impurities ($\vec{F}_{\text{coll},zav}$) and, at low impact parameters, with the full nuclear ion charge ($Z_{0,j}$ ($\vec{F}_{\text{coll},zo}$)), together with the corresponding Coulomb logarithms, $\ln\Lambda_{ej}$, should be considered. In such a case, the collision drag forces can be written:

$$\vec{F}_{\text{coll},e} = \vec{F}_{\text{coll},f} + \vec{F}_{\text{coll},b} = -\alpha_e \frac{e^4 m_e n_{ef} \ln\Lambda \gamma (\gamma + 1)}{4\pi\varepsilon_0 p^3} \vec{p}$$

$$\vec{F}_{\text{coll},i} = \vec{F}_{\text{coll},H} + \vec{F}_{\text{coll},zav} + \vec{F}_{\text{coll},zo} = -\alpha_e \frac{e^4 m_e n_{ef} Z_{\text{coll}} \ln\Lambda \gamma}{4\pi\varepsilon_0 p^3} \vec{p}$$  (3)

where

$$\alpha_e = \frac{n_{ef} \ln\Lambda_{ef} + n_{eb} \ln\Lambda_{eb}}{n_{ef} \ln\Lambda}; \quad Z_{\text{coll}} = \frac{n_H \ln\Lambda_{eH} + \sum_j n_{ej} \left(Z_{av,j} \ln\Lambda_{eavj} + Z_{0,j}^2 \ln\Lambda_{ezoj}\right)}{n_{ef} \ln\Lambda_{ef} + n_{eb} \ln\Lambda_{eb}}$$  (4)

($n_{ej}$ is the density of impurity $j$). Therefore, the resulting collision terms are increased by a factor $\alpha_e$ and $Z_{\text{coll}}$ is replaced by $Z_{\text{coll}}$.

The evaluation of $\ln\Lambda$ for relativistic electrons can be made following Ref. [2] and it is illustrated in Fig. 1 (left) which shows, for given JET-like parameters during disruptions ($B_0 = 3$ T; $R_0 = 3$ m; $n_H = 5 \times 10^{19}$ m$^{-3}$; $T_e = 10$ eV), the Coulomb logarithms vs the relativistic electron gamma factor, $\gamma$, for a plasma with a density of Xe ($Z = 54$) $n_x = 10^{20}$ m$^{-3}$. Typically, the value of $\ln\Lambda$ for collisions with bound electrons is $\sim 0.5 - 0.7$ times its value for collisions with free electrons and, similarly, $\ln\Lambda$ for collisions with the full and electron-shielded nuclear charge is $\sim 0.5 - 0.7$ times its value for collisions with
plasma protons. Nevertheless, if the impurity density is large enough and the impurities are weakly ionized, \( Z_{av} \ll Z_0, \ n_{cb} \approx n_z (Z_0 - Z_{av}) \gg n_{ef} \approx n_z Z_{av} \), so that the collisions with bound electrons and the full nuclear charge may become dominant. Hence, under the conditions of Fig. 1, the collision terms should be multiplied by a factor \( \alpha_e \) up to \( \sim 10 \) and \( Z_{eff} (\sim 4.6) \) should be replaced by \( Z_{coll} \sim 45 - 50 \) (Fig. 1 to the right).

3. Runaway Dynamics

Summarizing, the runaway dynamics in plasmas with high impurity content will be governed by the same set of equations than those in low impurity concentration plasmas but multiplying the collision terms by the factor \( \alpha_e \) and replacing \( Z_{eff} \) by \( Z_{coll} \) [Eq. (4)]. A first consequence is the increase in the threshold electric field for runaway generation as a result of the collisions with the bound electrons and the scattering by the full nuclear charge. In the relativistic limit, the drag force, \( F_{coll} \), has a limit, when \( v \to c \), and runaways will not appear if

\[
E_{||} \leq E_R = \frac{\min(F_{coll})}{e} = \alpha_e E_{RF} = \frac{e^3 (n_{ef} \ln \Lambda_{ef} + n_{cb} \ln \Lambda_{cb})}{4 \pi \varepsilon_0^2 m_e c^2}
\]  

where \( E_{RF} = \frac{e^3 n_{ef} \ln \Lambda}{4 \pi \varepsilon_0^2 m_e c^2} \) is the threshold electric field calculated using the density and Coulomb logarithm for free electrons. Hence, the threshold electric field, \( E_R \), is increased by a factor \( \alpha_e \) because of the collisions with partially stripped impurity ions.

A more accurate estimate can be obtained from an analysis using a simple test particle description of the runaway dynamics including the acceleration in the electric field, collisions with the electron and ion species, and synchrotron radiation losses [3]. The essential features of the phase-space structure of the test relaxation equations are those described in Ref.[3]. Two singular points exist in momentum space with a well-defined physical meaning: a saddle point, providing an estimate of the critical energy for runaway generation, and a stable focus, which gives the limiting energy that these runaways can achieve. This is illustrated in Fig. 2 which shows, for the same parameters than in Fig. 1, the electric field, \( E_{||} \), normalized to \( E_{RF} \), vs the electron energy, \( \gamma_s \), at the singular

![Figure 2: Normalized electric field (to \( E_{RF} \)) vs \( \gamma_s \) at the singular points with and without including the effect of the collisions with partially stripped impurity ions (full and dashed lines, respectively). Plasma parameters are the same than in Fig. 1.](image-url)
points. The full and dashed lines show the results with and without including the effect of collisions with the bound electrons and the full impurity nuclear charge, respectively. For a given electric field, branch I in the figure provides the critical energy for runaway generation while branch II gives the maximum attainable runaway energy. The minimum of $E_{\parallel}$ vs $\gamma_s$ provides the threshold electric field for runaway generation, $E_R$, including the effect of the radiation losses [3]. It is observed that, because of the collisions with partially stripped impurity ions, not only the threshold field, $E_R$, is substantially increased (by a factor $\sim \alpha_e$) but also, for a given electric field, the critical electric field for runaway generation (branch I) increases and the maximum energy than can be achieved by the generated runaway electrons (branch II) is noticeably reduced.

As an example of application, Fig. 3 illustrates the possibility of achieving runaway suppression by MGI of Ar, Kr and Xe ($Z = 18, 36, 54$) in a 5 MA JET-like disruption ($B_0 = 3$ T; $R_0 = 3$ m; $n_H = 5 \times 10^{19}$ m$^{-3}$). The figure shows the predicted ratio, $E_R/E_{\parallel}$, of the threshold field to the electric field during the disruption vs the impurity density, $n_z$. The electron temperature and the electric field during the disruption are self-consistently estimated from the power balance, $E_{\parallel}^2/\eta = n_e n_z L_z(T_e)$, where $L_z(T_e)$ is the impurity radiative cooling rate, and $n_e \approx n_H + Z_{av} n_z$. Runaway suppression ($E_R/E_{\parallel} \geq 1$) is achieved for Kr and Xe injection for $n_z \geq 10^{22}$ m$^{-3}$. Ar injection is less efficient, mainly because of the lower number of bound electrons. In the case of Kr, even if the number of bound electrons is lower than in Xe, the runaway suppression efficiency is similar as the Kr radiative cooling rate is smaller and, hence, the electron temperature will be larger, decreasing the value of the electric field during the disruption.

![Graph showing $E_R/E_{\parallel}$ vs $n_z$ for Ar, Kr, and Xe](image)

Figure 3: Predicted ratio $E_R/E_{\parallel}$ vs $n_z$ for Ar, Kr and Xe injection in a 5 MA JET-like disruption ($B_0 = 3$ T; $R_0 = 3$ m; $n_H = 5 \times 10^{19}$ m$^{-3}$).

References