Dissipative modulated electrostatic solitary plasma structures in the presence of a superthermal component

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Abstract

Focusing on modulated electron-acoustic plasma waves in superthermal plasmas, a nonlinear Schrödinger equation with complex coefficients (otherwise known as the Complex Ginzburg-Landau equation) is derived, by including an ad hoc dissipative term in the fluid equation of motion. The existence of dissipative solitons is discussed, and the effect of the supertherthermal electron component is investigated.

1. Introduction. Nonlinear amplitude modulation of electrostatic wavepackets in plasmas is a widely studied phenomenon, usually occurring due to the nonlinear self interaction of the carrier wave or to wave-plasma interactions. This mechanism is usually studied via a multiple space and time scale technique, leading to a nonlinear Schrödinger equation (NLSE), which possesses exact solutions in the form of envelope structures occurring when the spreading effect of dispersion and the self-steepening effect of nonlinearity balance each other. In the presence of dissipation (wave damping), the NLSE takes the form of a Complex Ginzburg-Landau equation (CGLE). Different types of solutions of the CGLE have been investigated, both in plasma physics [1,2,3] and in nonlinear optics [4,5,6].

In this study, we focus on a plasma containing superthermal particles modelled by a $\kappa-$ distribution [7,8]. We study the modulational properties of electron-acoustic (EA) wavepackets [7], by introducing dissipation via an *ad hoc* damping term in the fluid momentum equation.

2. Theoretical model. We consider a three component plasma consisting of inertial ("cold") electrons, $\kappa-$ distributed superthermal ("hot") electrons and stationary ions. The density of the superthermal ("*hot*") electrons is given by $n_h = n_{h0}\left(1 - \frac{e\phi}{(\kappa-3/2)k_B T_h}\right)^{-\kappa+1/2}$, where T_h is the characteristic (hot) electron temperature and the subscript "0" represents the unperturbed number density. The scaled fluid moment equations for the inertial (cold) electrons read:

$$\frac{\partial n}{\partial t} + \frac{\partial (nu)}{\partial x} = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = \frac{\partial \phi}{\partial x} - \nu u, \tag{2}$$

$$\frac{\partial^2 \phi}{\partial x^2} \approx \beta(n-1) + c_1\phi + c_2\phi^2 + c_3\phi^3, \tag{3}$$

where the fluid velocity u, the density n and the electrostatic potential ϕ are normalised to the EA speed $v_0 = (k_B T_h/m_e)^{1/2}$, to the unperturbed density of the inertial component n_{c0} and to $k_B T_h/e$, respectively. Time and space are scaled by $\omega_{ph}^{-1} = (4\pi n_{h0} e^2/m_e)^{-1/2}$ and by $\lambda_{D,h} = \left(k_B T_h/4\pi n_{h0} e^2\right)^{1/2}$. The phenomenological damping term $\nu = \nu_c/\omega_{ph}$, arises due to inter-particle collisions (char. frequency ν_c). We have defined the cold-to-hot electron density ratio $\beta = n_{c0}/n_{h0}$ and the $\kappa-$related coefficients: $c_1 = \frac{\kappa-1/2}{\kappa-3/2}$, $c_2 = \frac{c_1(\kappa+1/2)}{2(\kappa-3/2)}$, $c_3 = \frac{c_2(\kappa+3/2)}{3(\kappa-3/2)}$.

3. Modulated-amplitude electrostatic solitary excitations. Anticipating solitary excitations, we adopt a multiple scales technique [9,10]. We consider small ($\varepsilon \ll 1$) deviations of all state variables, say S $(= n, u, \phi)$, from the equilibrium state as $S = S^{(0)} + \Sigma_{n=1}^{\infty} \varepsilon^n S_n$. Secondary harmonic generation is accounted for via the ansatz $S_n = \Sigma_{l=-\infty}^{\infty} S_l^{(n)}(X,T) e^{il(kx-\omega t)}$; the reality condition $S_{-l}^{(n)} = S_l^{(n)*}$ is met by all state variables. All the perturbed states depend on the fast scales via the phase $\theta_1 = kx - \omega t$ only, while the slow scales only enter the $l-$th harmonic amplitude $S_l^{(n)}$.

Linear dynamics: The first order (linear) expressions provide the *dispersion relation*

$$\omega(\omega + i\nu) = \frac{\beta k^2}{k^2 + c_1} \, , \tag{4}$$

or

$$\omega = -\frac{i\nu}{2} + \sqrt{\frac{\beta k^2}{k^2 + c_1} - \frac{\nu^2}{4}} \, , \tag{5}$$

(provided that $k \geq (\frac{c_1 \nu^2}{4\beta - \nu^2})^{1/2}$ - otherwise overdamping occurs; recall that $c_1 > 0$). The 1st-order perturbation amplitudes are: $u_1^{(1)} = \frac{\omega}{k} n_1^{(1)} = -\frac{\omega(k^2+c_1)}{\beta k} \phi_1^{(1)}$.

Nonlinear envelope description: The 2nd order equations provide the compatibility condition: $\frac{\partial \phi_1^{(1)}}{\partial T_1} + v_g \frac{\partial \phi_1^{(1)}}{\partial X_1} = 0$ along with the corresponding 0th, 1st and 2nd harmonic amplitudes (to $\sim \varepsilon^2$). Interestingly, the group velocity is real-valued – cf. (5) – yet $\nu-$dependent. Annihilating secular terms in ε^3, we obtain a damped NLSE in the form:

$$i\frac{\partial \psi}{\partial \tau} + P\frac{\partial^2 \psi}{\partial \zeta^2} + Q_r \mid \psi \mid^2 \psi + iQ_i \mid \psi \mid^2 \psi = 0 \, , \tag{6}$$

where $\psi = \phi_1^{(1)}$ and the (slow) independent variables are $\zeta = \varepsilon(x - v_g t)$ and $\tau = \varepsilon^2 t$.

P and $Q(= Q_r + iQ_i)$ are dispersion and nonlinearity coefficients respectively. The loss term (involving the imaginary part Q_i) arises due to damping (and cancels in the limit $\nu \to 0$).

Damped envelope solitons: Since there is no gain term (i.e., no imaginary part of P), coherent propagation of stable Perreira-Stenflo [2-3] or Akhmediev [4] solitons cannot be sustained in

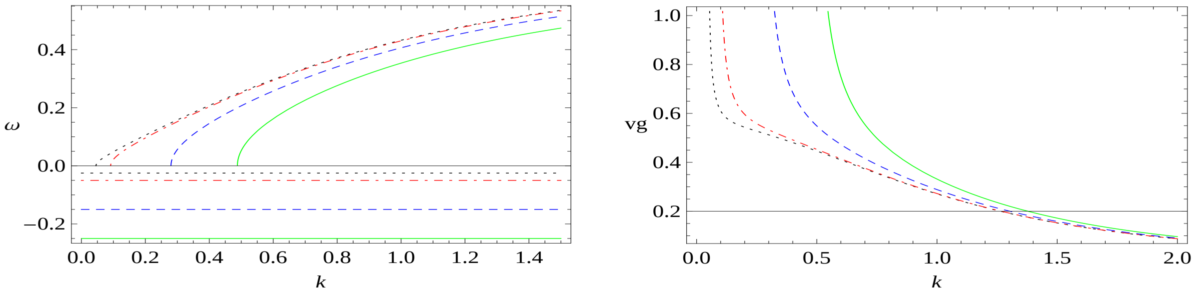

Figure 1: Variation of the frequency ω and group velocity v_g with wavenumber k, for different values of ν; here $\nu = 0.05$ (black curve), 0.1 (red curve), 0.3 (blue curve), 0.5 (green curve) and $\kappa = 4$, $\beta = 0.5$. We have chosen high values of ν, to emphasize the qualitative effect.

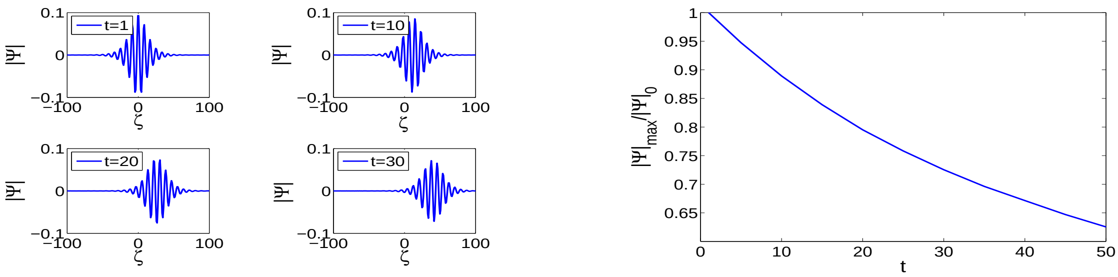

Figure 2: Bright type solitary structures (for $PQ_r > 0$): time evolution (left panel) and variation of maximum amplitude (right panel). Arbitrary parameter values; here, $P = Q_r = 1$ and $Q_i = 2$.

this model. Eq. (6) is thus to be viewed as a dissipative NLSE, accounting for envelope soliton solutions [7] which are damped, i.e., they decay in time and space. This is obvious in Figures 2-3. The decay rate is expected to be related to the value of Q_i, which may be quite high (order ~ 1 or higher), even for small values of ν: see Fig. 4b. On the other hand, the damping effect on the P/Q_r ratio (Fig. 4a) is not dramatic, in that higher ν values only slightly increase P/Q_r (which affects the envelope soliton width [7]) for large wavelengths (small k) (yet bear almost no effect for higher k; see the right part of Fig. 4a). We emphasize that EA waves survive Landau damping in a wide wavenumber region [7], hence the range of values considered in our plots.

4. Modulational instability. The modulational instability (MI) of electrostatic wavepackets can be investigated via a tedious algebraic calculation, omitted here for brevity (to be reported elsewhere in detail [11]). Summarizing those results, the “traditional" criterion ($PQ_r < 0$ for stability; MI otherwise [7]) is dramatically modified due to the damping term involving Q_i in (6). Although wavepacket decay cannot be avoided, as obvious (for $\nu \sim Q_i \neq 0$), it may be slowed down for a significant amount of time by wave growth due to modulational instability occurring in a wider parameter region (and even partly for $PQ_r < 0$ as well). This interplay be-

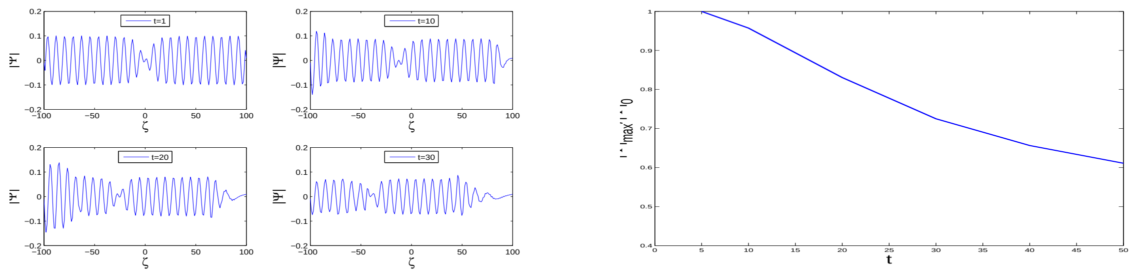

Figure 3: Dark-type solitary structures ($PQ_r < 0$): time evolution (left panel) and variation of maximum amplitude (right panel). Arbitrary parameter values; here, $P = -Q_r = 1$ and $Q_i = 2$.

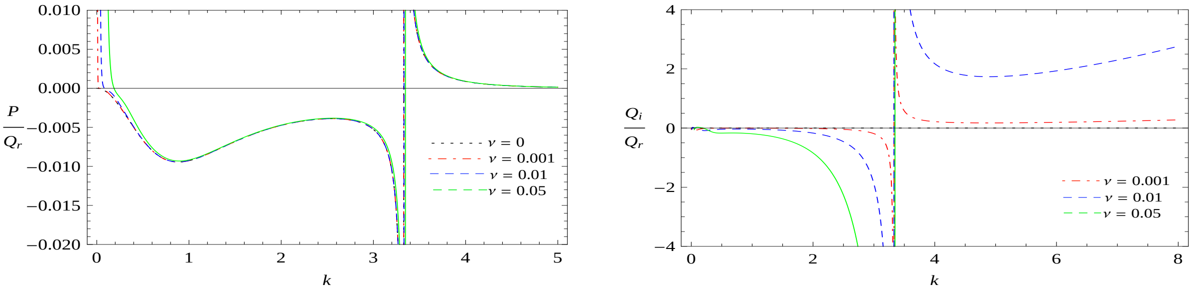

Figure 4: Variation of the P/Q_r ratio (left panel) and Q_i/Q_r ratio with carrier wavenumber k for different collision frequency ν; here $\kappa = 3$, $\beta = 0.5$.

tween amplitude growth (due to MI) and decay (due to damping) is clearly visible in numerical simulations, to be reported elsewhere [12]. A link is thus established with dissipative soliton theory, e.g., in nonlinear optics [4-6], to be explored further in future work.

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