Triple floating potential of an electron emitting electrode that is immersed in a plasma that contains thermal and mono-energetic electrons

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Introduction

Plasmas that in addition to the basic, usually Maxwellian electron population, contain also an energetic electron population are very important in technological and fusion applications. Energetic electron populations are often created in fusion devices during electron cyclotron and lower hybrid resonance heating and at the rf current drive. On the other hand sheath formation in front of electron emitting electrodes is also a very important topic for understanding emissive probe behavior. Emissive probes are a very important plasma diagnostic tool. In this work we study the potential formation in front of an electron emitting electrode immersed in a plasma that contains an isotropic mono-energetic electron beam.

Model

An infinitely large planar electrode (collector) has its surface perpendicular to the x axis and is located at $x = 0$ [1]. This electrode absorbs all the particles that hit it. On the other hand it may also emit electrons. This electron emission can be thermal or secondary. The details of the emission mechanism are not essential for the model and in this work it will not be specified. When the collector is floating or biased negatively with respect to the plasma potential, it reflects negative electrons and attracts positive ions. The potential profile in the sheath is determined by a one-dimensional Poisson equation:

$$\frac{d^2\Phi}{dx} = -\frac{e_0}{\varepsilon_0}\left(n_i(x) - n_1(x) - n_2(x) - n_3(x)\right). \qquad (1)$$

The meaning of the symbols is the following: Φ is the potential, e_0 is the elementary charge, ε_0 is the permittivity of the free space, n_i is the density of the singly charged positive ions, n_1 is the density of the bulk electron population, n_2 is the density of the beam or primary electrons and n_3 is the density of the emitted electrons. The potential very far away from the collector is set to zero $\Phi(x \to \infty) = 0$. The collector potential is Φ_C and it is negative. As one approaches to the collector from the plasma, the potential slowly decreases and a pre-sheath is formed. This is a region, where the plasma is still quasi-neutral but a weak electric field exists, which accelerates positive ions towards the collector and negative electrons in the opposite direction. The length scale of the pre-sheath is L and it is determined by some characteristic binary process in the plasma. At the distance $x = d$ from the collector the plasma quasi-neutrality breaks down and a sheath with an excess of positive space charge is formed. The plane at $x = d$ is called the sheath edge. The potential there is Φ_S and this is the last point, where the quasi neutrality is still valid. Note that Φ_S is also negative with $\Phi_C < \Phi_S$.

The ion density in the sheath is found from the energy and flux conservation arguments and it is given by: $n_i(x) = n_S\sqrt{\frac{e_0\Phi_S+A_c}{e_0\Phi(x)}}$. Here A_c is the energy that the ions loose in the pre-sheath because of collisions and n_S is the ion density at the sheath edge. The density of the main electron population is given by the Boltzmann law: $n_1(x) = n_1\exp\left(\frac{e_0\Phi(x)}{kT}\right)$. Here T is the electron temperature, k is the Boltzmann constant and n_1 is the density of the bulk electron population at a large distance from the collector, where the potential is zero. The beam electrons are assumed to be mono-energetic. At a large distance ($x > L$) from the collector their density is n_2 they have all the same speed v_2. The directions of their velocities however, are uniformly distributed in space. The velocity distribution is given by:

$$f_2(v) = \frac{n_2}{4\pi v_2^2}\delta(v - v_2). \tag{2}$$

The density of the beam electrons at the distance x from the collector is found by integration of the distribution (2) over velocity space: $n_2(x) = \int_v f_2(v)d^3v = \frac{1}{2}n_2\left(1-\sqrt{-\frac{2e_0\Phi(x)}{m_e v_2^2}}\right)$. The density of the emitted electrons in the sheath is also found from the flux and energy conservation arguments: $n_3(x) = j_3/\sqrt{v_C^2 - \frac{2e_0(\Phi_C-\Phi(x))}{m_e}}$. The flux of the emitted electrons from the collector j_3 is assumed to be a given parameter and v_C is the initial velocity of the emitted electrons at the collector. When the particle densities are inserted into (1) and the quasi-neutrality condition at the sheath edge is taken into account, the Poisson equation (1) is written as:

$$\begin{aligned}\frac{d^2\Psi}{dz^2} &= \exp(\Psi(z)) + \frac{\beta}{2}\left(1-\sqrt{-\frac{2\Psi(z)}{\vartheta^2}}\right) + J_3\left(\Omega^2 - 2(\Psi_C-\Psi(z))\right)^{-\frac{1}{2}} - \\ &-\sqrt{\frac{\Psi_S}{\Psi(z)}}\left(\exp(\Psi_S) + \frac{\beta}{2}\left(1-\sqrt{-\frac{2\Psi_S}{\vartheta^2}}\right) + J_3\left(\Omega^2-2(\Psi_C-\Psi_S)\right)^{-\frac{1}{2}}\right).\end{aligned} \tag{3}$$

The following variables have been introduced:

$$\begin{aligned}&\Psi = \frac{e_0\Phi(x)}{kT},\ \Psi_C = \frac{e_0\Phi_C}{kT},\ \Psi_S = \frac{e_0\Phi_S}{kT},\ \varphi = \frac{A_c}{kT},\ \mu = \frac{m_e}{m_i},\ J_3 = \frac{j_3}{n_1\sqrt{\frac{kT}{m_e}}},\\ &J_t = \frac{j_{et}}{e_0 n_1\sqrt{\frac{kT}{m_e}}},\ \beta = \frac{n_2}{n_1},\ v_C = \Omega\sqrt{\frac{kT}{m_e}},\ v_2 = \vartheta\sqrt{\frac{kT}{m_e}},\ z = \frac{x}{\lambda_D},\ \lambda_D = \sqrt{\frac{\varepsilon_0 kT}{n_1 e_0^2}}.\end{aligned} \tag{4}$$

With these variables the total current density to the collector is written in the following form:

$$\begin{aligned}J_t &= \frac{1}{\sqrt{2\pi}}\exp(\Psi_C) + \frac{1}{4}\beta\vartheta\left(1+\frac{2\Psi_C}{\vartheta^2}\right)H\left(1+\frac{2\Psi_C}{\vartheta^2}\right) - J_3 - \\ &-\sqrt{-2\mu(\Psi_S+\varphi)}\left(\exp(\Psi_S) + \frac{\beta}{2}\left(1-\sqrt{-\frac{2\Psi_S}{\vartheta^2}}\right) + J_3\left(\Omega^2-2(\Psi_C-\Psi_S)\right)^{-\frac{1}{2}}\right),\end{aligned} \tag{5}$$

When the Poisson equation (3) is multiplied by $d\Psi/dz$ and integrated once over Ψ, one half of the square of the electric field $g(\Psi)$ in the sheath is obtained as a function of the potential Ψ:

$$\frac{1}{2}\left(\frac{d\Psi}{dz}\right)^2_\Psi - \frac{1}{2}\left(\frac{d\Psi}{dz}\right)^2_{\Psi=\Psi_S} = \frac{1}{2}\left(\frac{d\Psi}{dz}\right)^2_\Psi = \exp(\Psi) - \exp(\Psi_S) +$$
$$+J_3\left(\sqrt{\Omega^2 - 2(\Psi_C - \Psi)} - \sqrt{\Omega^2 - 2(\Psi_C - \Psi_S)}\right) +$$
$$+\frac{\beta}{6\vartheta}\left(2\sqrt{2}\left(\Psi_S\sqrt{-\Psi_S} + (-\Psi)^{\frac{3}{2}}\right) - 3\vartheta\left(\Psi_S - \Psi\right)\right) + \left(\Psi_S + \sqrt{\Psi\Psi_S}\right)\left(\beta + 2\exp(\Psi_S)\right) +$$
$$+\left(\frac{\beta\sqrt{2}}{\vartheta}\Psi_S + 2J_3\left(\Omega^2 - 2(\Psi_C - \Psi_S)\right)^{-\frac{1}{2}}\right)\left(\sqrt{-\Psi} - \sqrt{-\Psi_S}\right) \equiv g(\Psi).$$

Note that in the asymptotic two-scale limit of our model ($L \gg d \gg \lambda_D$) the electric field at the sheath edge is zero. If the electron emission is space-charge limited or critical, the following condition is fulfilled:

$$g(\Psi = \Psi_C) = 0. \tag{6}$$

If a stable sheath is to be formed the positive ions must enter the sheath with a certain minimum velocity, called the ion sound velocity, $v_S \geq c_S$. This is known as the Bohm [2] criterion. Using the variables (4) and with equality sign, the Bohm criterion gets a form of a transcendental equation for the sheath edge potential Ψ_S:

$$\Psi_S = \left(-\frac{1}{2}\right)\frac{\exp(\Psi_S) + \frac{\beta}{2}\left(1 - \sqrt{-\frac{2\Psi_S}{\vartheta^2}}\right) + J_3\left(\Omega^2 - 2(\Psi_C - \Psi_S)\right)^{-\frac{1}{2}}}{\exp(\Psi_S) + \frac{\beta}{2\vartheta\sqrt{-2\Psi_S}} - J_3\left(\Omega^2 - 2(\Psi_C - \Psi_S)\right)^{-\frac{3}{2}}} - \varphi \equiv f(\Psi_S). \tag{7}$$

Results

We now use the model to calculate the current voltage characteristics of an emissive probe in a typical low pressure hot cathode discharge plasma machine. The following parameters are selected: μ = 1/(40·1836) $\approx$ 1.36·10^{-5} (argon ions), Ω = 0.001 and φ = 0, while β, ϑ and J_3 are varied and are indicated in Figure 1. If J_3 is increased one expects that the floating potential Ψ_f will increase (become less negative) if the under parameters are not changed. The floating potential Ψ_f can be found by solving the system of equations (5) with J_t = 0 and (7) for Ψ_f and Ψ_S. In the bottom right plot of Figure 1 the floating potential is shown as a function of J_3 for μ =1.36·10^{-5}, Ω = 0.001 and φ = 0. For one curve β = 0 is selected and for other 2 curves we put β = 0.01 and 2 values of ϑ, which are ϑ = 6 and ϑ = 9. The presence of electron beam decreases Ψ_f considerably.

If J_3 increases, the absolute value of electric field at the collector decreases and eventually drops to zero at some J_3. The emission becomes space charge limited or critical. For smaller J_3 the emission is called temperature limited - implying that the predominant mechanism of electron emission is Richardson emission. The collector potential at which (for a given J_3) the transition changes from space charge limited into temperature limited emission is labeled Ψ_{C0}.

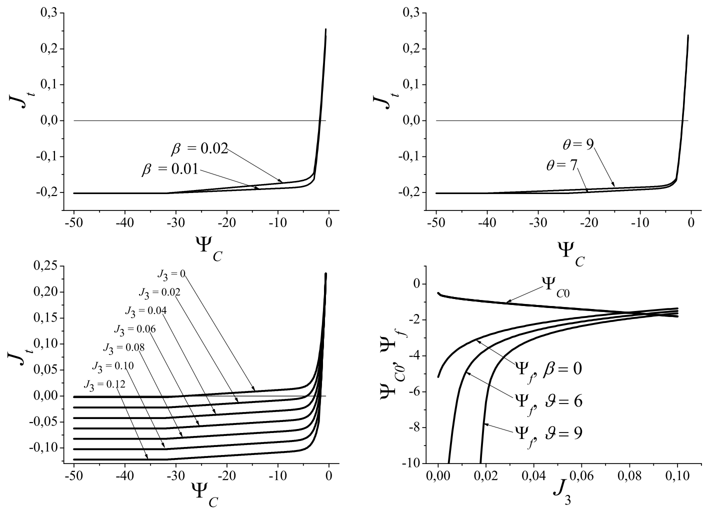

Figure 1: Current voltage characteristics, transition potentials Ψ_{C0} and floating potentials Ψ_f for various β, ϑ and J_3.

For given values of μ, β, ϑ, φ, Ω and J_3 it can be found easily by solving the system of equations (6) and (7) for Ψ_S and Ψ_{C0}. Dependence of Ψ_{C0} on J_3 for μ =1.36·10^{-5}, Ω = 0.001, φ = 0, β = 0 and for β = 0.01 with 2 values of ϑ, ϑ = 6 and ϑ = 9 is also shown in the bottom right plot of Figure 1. The beam density and energy β and ϑ have almost no effect to the dependence of Ψ_{C0} on J_3 and the 3 curves can not be distinguished on the plot. As J_3 increases Ψ_{C0} decreases and Ψ_f increases and eventually they become equal. This explains the "saturation" of the floating potential of an emissive probe with increased emission. When Ψ_{C0} exceeds the Ψ_f the probes floating potential can not come closer to the plasma potential (in our model zero) any more. So when one calculates the current voltage characteristics for given values of μ, β, ϑ, φ, Ω and J_3 first the transition potential Ψ_{C0} has to be found. Then for every $\Psi_C \leq \Psi_{C0}$ the Ψ_S has to be found from (7) and then J_t is found from (5). For $\Psi_C > \Psi_{C0}$ the system (6) and (7) must be solved for Ψ_S and the critical emission J_{3cr}, which are then both inserted into (5) to find J_t. In the bottom left graph the current voltage characteristics are shown for μ =1.36·10^{-5}, Ω = 0.001, φ = 0, β = 0.01, ϑ = 8 and several J_3. In the top graphs the effect of β and ϑ to the characteristics is illustrated. For the top left graph the parameters are: μ =1.36·10^{-5}, Ω = 0.001, φ = 0, ϑ = 8, J_3 = 0.2, while for the top right graph the parameters are: μ =1.36·10^{-5}, Ω = 0.001, φ = 0, β = 0.01 and J_3 = 0.2.

References

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