Conformal coordinates for turbulence computations on shaped tokamak geometries

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Introduction: The magnetised nature of the tokamak plasma yields a strong spatial scale separation between the directions along (parallel) and across (perpendicular) the equilibrium magnetic field. The use of field aligned coordinates is therefore a common practice in turbulence computations due to its numerical efficiency in treating the parallel dynamics. This comes at a price for strongly shaped tokamak magneto-hydro-dynamic (MHD) equilibria, which is typically the situation at the plasma edge region, where special features like an X-point may be present. A correct representation of the perpendicular dynamics at finite resolution may not be possible unless specific counter-measures are devised to compensate for the inherent rapid variation of the field aligned coordinate systems' metric coefficients along the field lines, which translates into grid mesh deformation. Here a novel geometrical treatment to address this issue is discussed. A consistent combination of magnetic field aligned and conformal coordinates [1] is used together with a shifted metric procedure [2]. This allows an efficient treatment of the dynamics along the magnetic field lines while enforcing the necessary isotropicity in the plane perpendicular to it at the grid spacing level, best representing the turbulence physical properties.

Geometry theory: Any globally magnetic field aligned coordinate system must be constructed using generalised poloidal (ϑ) and toroidal (ζ) angles that yield a straight representation of the field lines. In this case, the magnetic field pitch becomes a flux label, $q = q(x)$ and a Clebsch representation of the magnetic field is possible $\mathbf{B} = B^s\sqrt{g}\nabla x \times \nabla y$, where $g^{-1/2} = \nabla x \times \nabla y \cdot \nabla s$ is the coordinate system Jacobian and s the parallel angle coordinate. The remaining coordinates x, y are a flux label and an angle, respectively, that represent the perpendicular plane. Either the ϑ or ζ can be used for s, the choice depending only on the combination used with them and q to define y. Because the magnetic field is sheared, the non-orthogonality between the x and y coordinates (g^{xy}) varies along the field lines according to

$$g^{xy} = \nabla x \cdot \nabla \left(\alpha(x)\vartheta - \frac{\alpha(x)}{q(x)}\zeta \right) = \left(\alpha(x) g^{x\vartheta} - \frac{\alpha(x)}{q(x)} g^{x\zeta} \right) + \left(\vartheta\alpha' + \zeta \frac{q'\alpha - q\alpha'}{q^2} \right) g^{xx} \quad (1)$$

with the notation $(') = \partial/\partial x$ and the choice of $\alpha = 1$ for $s = \zeta$ and $\alpha = q$ for $s = \vartheta$. The local and global magnetic shear components can be identified with the parallel partial derivative of terms in the first and second parenthesis on the rhs, respectively. The latter has a secular dependence along the field lines, which has been shown to introduce numerically artificial ballooning character into the dynamics via its effect on the perpendicular laplacian operator at finite resolutions. The shifted metric procedure [2] addresses the issue by applying rigid shifts in the perpendicular angle to minimise g^{xy} at discrete positions along the field lines, with the shifts being consistently accounted for in the parallel derivatives. Even with the shear deformation $d \equiv g^{xy}/g^{xx}$ problem solved, there is still grid cell deformation associated to the magnetic flux expansion. The best way to illustrate it is to note that on Clebsch coordinates $B^2 \approx (\sqrt{g}B^s)^2 \left[g^{xx} g_k^{yy} \right]$ is inversely proportional to the perpendicular coordinate volume element, where the k subscript indicates that the shifted metric procedure was used. If one of the contravariant metric elements varies faster than B^2, then the remaining one varies inversely. The faster the variation is, the bigger stretch/squeeze deformation results. This is a inherent property of all Clebsch coordinate systems, and suggests the introduction of a measure of the grid cell isotropy given by $R_c \equiv g_k^{yy}/g^{xx}$ (conformal ratio). An isotropic grid has therefore a (normalised) value of $R_c = 1$. The amplitude of the departure from this situation gives an estimate of the increase in the grid-count that would be needed to achieve everywhere the same isotropic resolution yield where $R_c = 1$. Fig. 1(left) shows R_c for Hamada (Clebsch) coordinates at the edge of a typical ASDEX Upgrade equilibrium. Values ~ 40 are reached, which explains the need to address the issue. The conformal tokamak geometry [1] constitutes such an attempt. It consists of using both a Clebsch and a non-Clebsch coordinate system mapped together in a simple way. The former, being field aligned, treats the parallel dynamics at low computational cost, while the latter ensures an highly isotropic perpendicular grid. The symmetry (PEST) coordinates (ψ, θ, ϕ) provide the simplest choice of Clebsch coordinates for an axisymmetric tokamak magnetic field, where $\Psi = -2\pi\psi$ is the poloidal magnetic flux, ϕ is the geometrical toroidal angle and θ is the generalised poloidal angle which yields a flux label field pitch $q = q(\psi)$. The non-Clebsch system is defined to use the same toroidal coordinate, such that the map between both systems reduces to the two-dimensional poloidal plane. The requirement is that the conformal volume element (dV) follow the flux surfaces' expansion both radially and toroidally. The Jacobian is then defined to be $(\sqrt{g_c})^{-1} \equiv g^{xx}/R$. The comparison of dV on both coordinate system yields

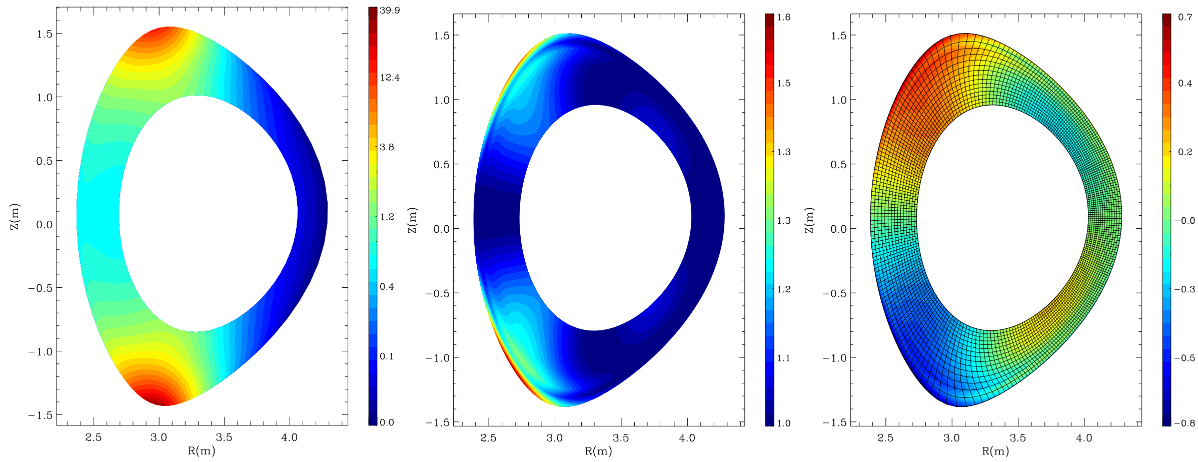

Figure 1: Conformal ratio for an ASDEX Upgrade based equilibrium computed with the HELENA MHD solver. The the radial domain shown is $0.65 < \rho_{\text{tor}} < 0.96$ in units of normalised toroidal magnetic flux for (left) Hamada coordinates and (center) confomal coordinates. (right) conformal grid and associated shear deformation d.

$(qR^2/I)d\theta = f(\psi)(R/g^{xx})dy$ and $f(\psi) \equiv -\partial x/\partial\psi$ where the symmetry coordinates' Jacobian $(\sqrt{g})^{-1} \equiv I/qR^2$ was used and the minus sign in f arises to ensure right-handiness. These expressions provide the map between (θ, ψ) and (x, y) and integrating them yields the conformal coordinates. Furthermore, comparing the definition for the conformal Jacobian with the determinant of the Jacobian matrix yields the following relation $g^{yy} = [(g^{xx})^2 + (g^{xy})^2]/g^{xx}$, which leads to the expression for the conformal ratio of this coordinates $R_c = g^{yy}/g^{xx} = 1 + d^2$. This shows that, barring exceptional cases, the conformal coordinates keep the conformal ratio close to unity, as seen in the middle frame of Fig.1. The same equilibrium was used before for the Clebsch coordinates which clearly suffer from strong conformal deformation, especially in the X-points' vicinity (note the different colour scales). The rightmost plot shows the shear deformation d for the conformal coordinates, which measures the nonorthogonality due to the y-dependent (poloidal) part of the local magnetic shear. The conformal grid itsel is also plotted.

Turbulence simulations: The GEMZ model solves 6-moment gyrofluid equations [3] on the conformal tokamak geometry described above. It has local parameters and global geometry (no flux tube approximation invoked). The main point of this section is to test if the new geometrical treatment works properly. To do so we run GEMZ on a suitable test case and make resolution tests, which means verifying if the same (qualitative) answer is obtained when the perpendicular grid-count is doubled. The plasma parameters chosen reflect a core case ($T = 2$KeV, $n = 3 \times 10^{19}\text{m}^{-3}$, $B = 3$T and $q = 4$) and the radial domain was placed in the edge, namely, at $0.65 < \rho_{\text{tor}} < 0.96$. The justification for the latter is obvious on the grounds of maximazing flux surface shaping. For the former, it is two fold: (i) larger gyroradius yield lower resolution cost requirements; (ii) core turbulence is weaker which is better if the model, like GEMZ, does not evolve the zonal thermodynamic profiles. To be consistent with the radial box size, the profile

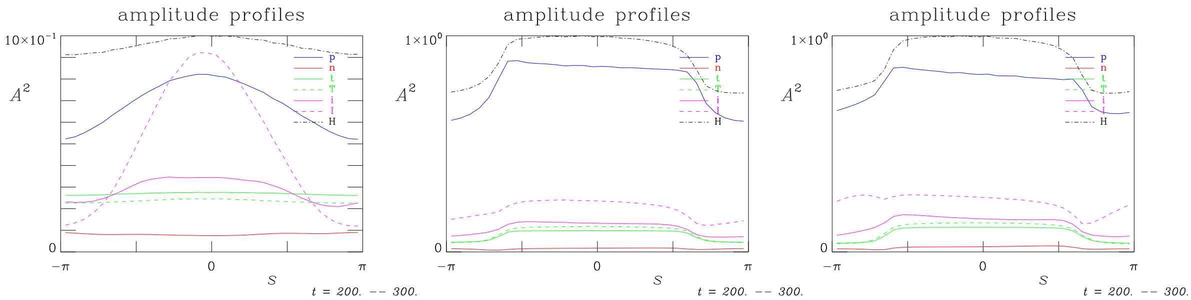

Figure 2: Squared amplitude envelope of the (blue) electrostatic potential, (red) electron density, (green) electron temperature, (magenta) ion temperature and (black) nonadiabatic density fluctuations along the parallel coordinate.

scale lengths were set to $L_x/L_n = Lx/L_T/2 = 1/4$. An initial simulation with cylinder geometry (no shaping) was performed on a typical half resolution (HR) perpendicular grid, which means half a grid node per sound gyroradius ρ_s. For the chosen radial domain this gives a grid count of $32 \times 512 \times 16$ in (x, y, s). The simulation was repeated on the HELENA equilibrium of Fig. 1, both with the same grid count, and with a doubled perpendicular grid count (FR), namely $64 \times 1024 \times 16$. The statistical analysis was performed during the turbulent saturated phase on all cases. Fig. 2 shows the parallel mode structure, which is found to be very different whether the shaped geometry is used or not. If it is used, a strong interchange (MHD) signature results, whereas a more ITG-like character is yield for the cylinder case. For this reason the former gives stronger turbulence levels, which was not expected *a priori*. The explanation lies in the double null X-point configuration of the HELENA equilibrium used, which seems to cause some degree of decoupling between the high (HFS) and low field sides (LFS) of the plasma (see sharp changes at $s \approx \pm\phi/2$ in Fig. 2). This results in interchange dominated dynamics. The remaining diagnostics (not shown here) corroborate this picture. The same qualitative results were obtained with the FR grid-count Fig. 2(right), showing that the system is being well resolved by the conformal geometry. Also noteworthy is that a similar physcial picture was observed using a scrape-off layer simulations with double limiter cuts on a cylinder geometry [4]. A comparative study with the GEMR, which employs the traditional Clebsch Hamada geometry on the same gyrofluid equations [3], is under way. The preliminary results obtained so far with GEMR on the same HELENA equilibrium show changes in the mode structure when the resolution test is performed, indicating that the simulations did not correctly represent the perpendicular dynamics at the finite resolutions used. This result is not surprising given the severe deformation yield for by this geometry, as shown in Fig. 1(left).

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